Amplifiers And Signal Processing

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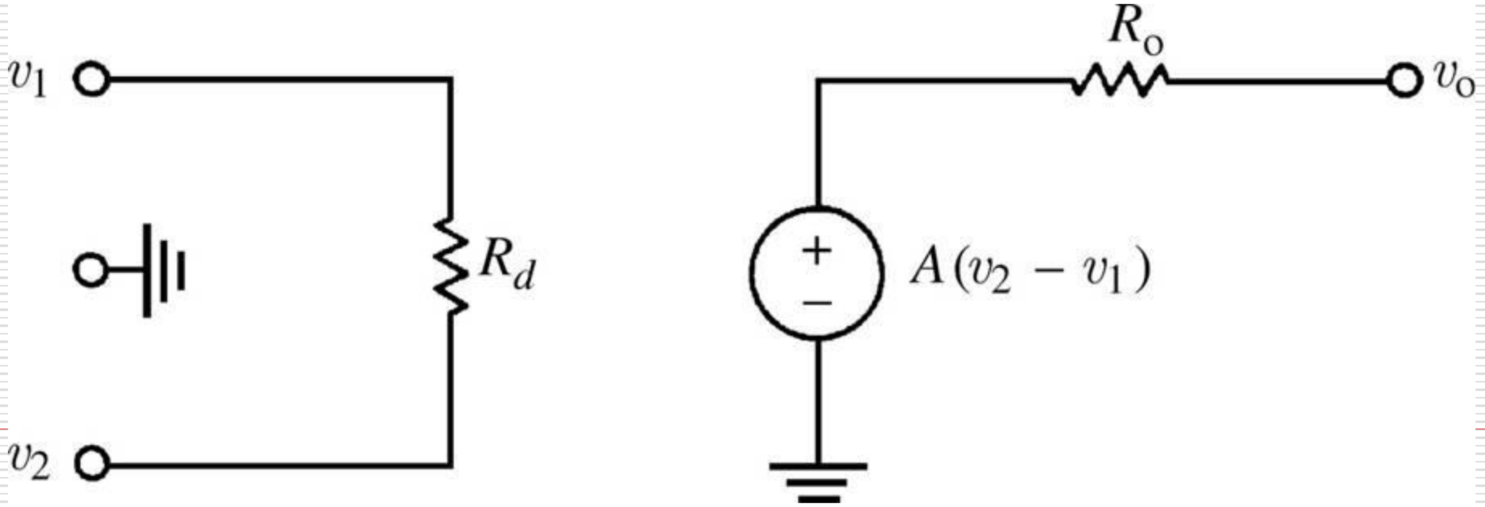
INTRODUCTION

- Most bioelectric signals are small and require amplification. Amplifiers are also used for interfacing sensors that sense body motions, temperature, and chemical concentrations.
- In addition to simple amplification, the amplifier may also modify the signal to produce frequency filtering or nonlinear effects.
- This course emphasizes the operational amplifier (op amp), which has revolutionized electronic circuit design.
- Most circuit design was formerly performed with discrete components, requiring laborious calculations, many components, and large expense.
- Now a 20 cents op-amp, a few resistors, and knowledge of Ohm's law are all that is needed.

- An op amp is a high-gain dc differential amplifier.
- It is normally used in circuits that have characteristics determined by external negative-feedback networks.
- The best way to approach the design of a circuit that uses op amps is first to assume that the op amp is ideal.
- After the initial design, the circuit is checked to determine whether the non-ideal characteristics of the op amp are important.
- If they are not, the design is complete; if they are, another design check is made, which may require additional components.

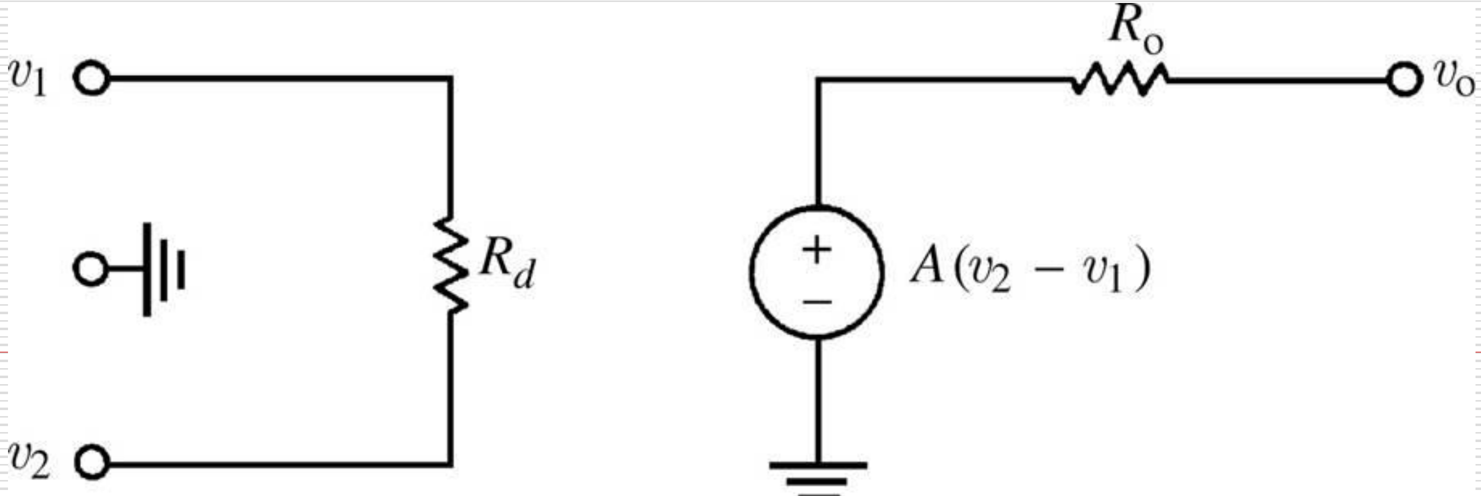
Equivalent Circuit For Non-ideal Op-amp

- The two inputs are v_1 and v_2.
- A differential voltage between them causes current flow through the differential resistance R_d.
- The differential voltage is multiplied by A, the gain of the op amp, to generate the output-voltage source.
- Any current flowing to the output terminal v_O must pass through the output resistance R_o.



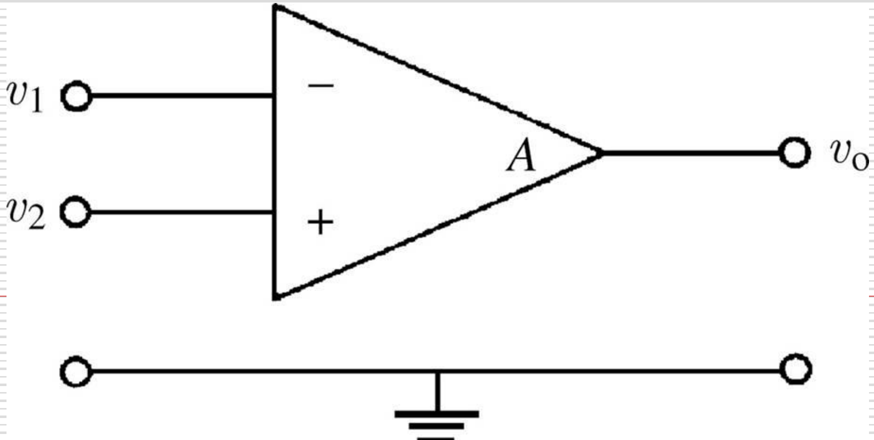
Equivalent Circuit For Non-ideal Op-amp

$A = \infty$ (gain is infinity): practical values 20k – 200k
$v_o = 0$, when $v_1 = v_2$ (no offset voltage)
$R_d = \infty$ (input impedance is infinity): CMOS op amps are close to ideal
$R_o = 0$ (output impedance is zero): practical values 20-100Ω
Bandwidth = ∞ (no frequency-response limitations) and no phase shift
Gain Bandwidth product: practical values ~MHz frequency where open-loop gain drops to 1.



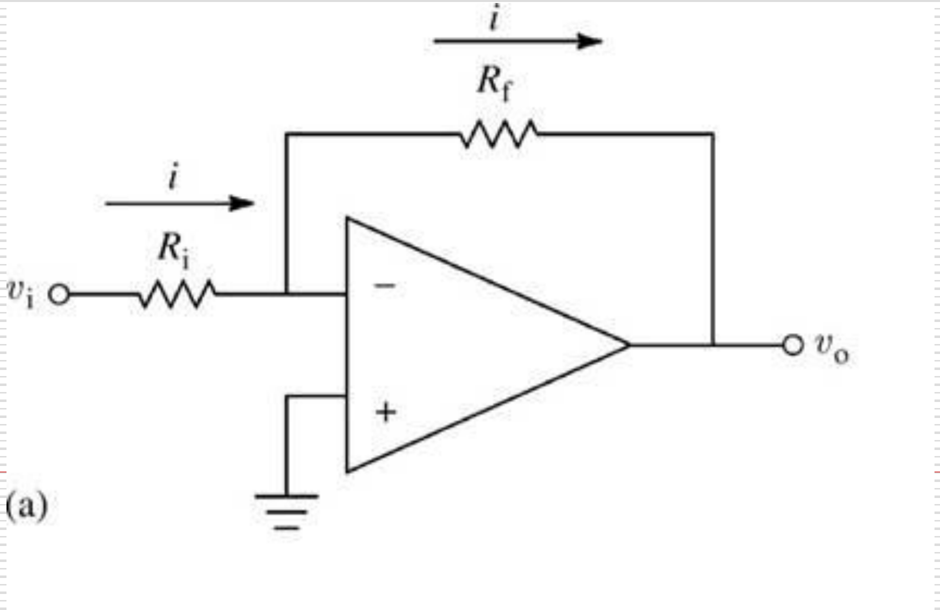
Two Basic Rules

- **RULE 1:** *When the op-amp output is in its linear range, the two input terminals are at the same voltage.*
  - *if the two input terminals were not at the same voltage, the differential input voltage would be multiplied by the infinite gain to yield an infinite output voltage. Actually the op-amp specifications guarantee a linear output range of only +10 V, although some saturate at about +13 V.*
- **RULE 2:** *No current flows into either input terminal of the op amp.*
  - we assume that the input impedance is infinity, and no current flows into an infinite impedance. Even if the input impedance were finite, Rule 1 tells us that there is no voltage drop across *Rd* so therefore, no current flows.



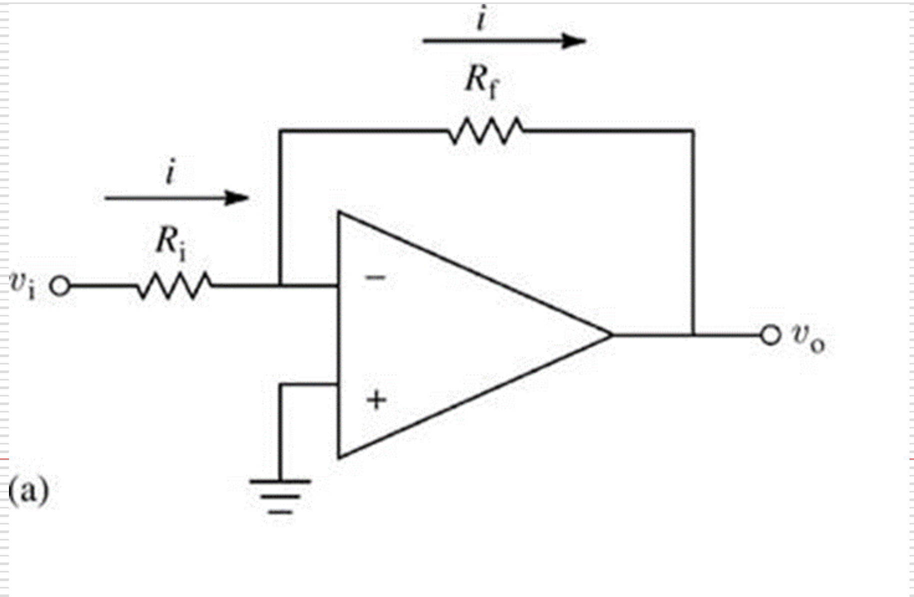
Inverting Amplifier

- It is widely used in **instrumentation amplifier** (*To be explained later*).
- v_O is fed back via R_f to the negative input of the op amp providing
  - increased bandwidth
  - lower output impedance
  - linearity
- Current flowing through the input resistor R_i also flows through the feedback resistor R_f.



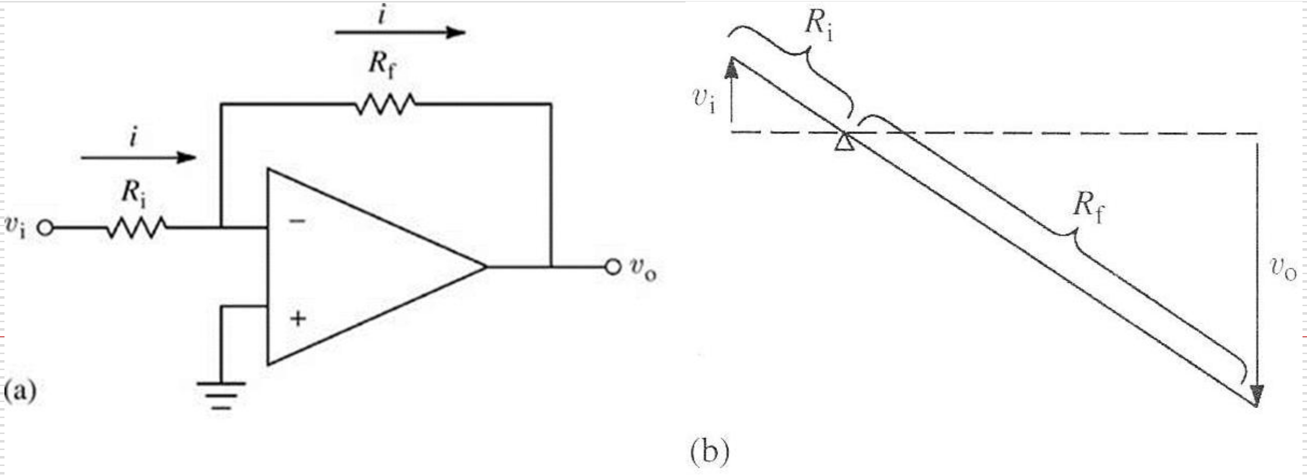
- **Rule 1** implies that positive input voltage=negative input voltage=0V, a condition known as a virtual ground.
- Because the right side of R_i, is at 0 V and the left side is v_I, by Ohm's law the current *i* through R_i, is $i = v_I / R_i$.
- By **Rule 2**, no current can enter the op amp; therefore *i* must also flow through R_f. This produces a voltage drop across R_f of iR_f since the left end of R_f is at 0 V, then

$$v_o = -iR_f = -v_i \frac{R_f}{R_i} \quad \text{or} \quad \frac{v_o}{v_i} = \frac{-R_f}{R_i}$$



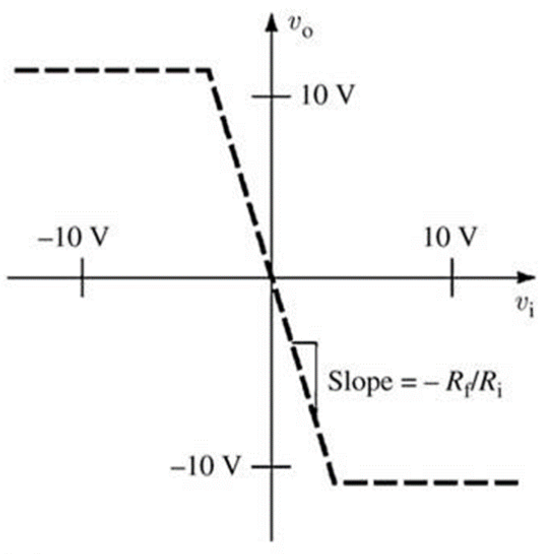
LEVER ANALOGY

- A lever is formed with arm lengths proportional to resistance values.
- Because the negative input is at 0 V, the fulcrum is placed at 0 V, as shown.
- If R_f is three times R_i, any variation of v_i results in a three-times-bigger variation of v_o.
- This circuit is a voltage-controlled current source (VCCS) where for any load R_f (Jung, 1986). The current i through Rf is v_i/R_i, so v_i controls *i*.
- Current sources are useful in electrical impedance plethysmography for passing a fixed current through the body.



INPUT OUTPUT CHARACTERISTIC

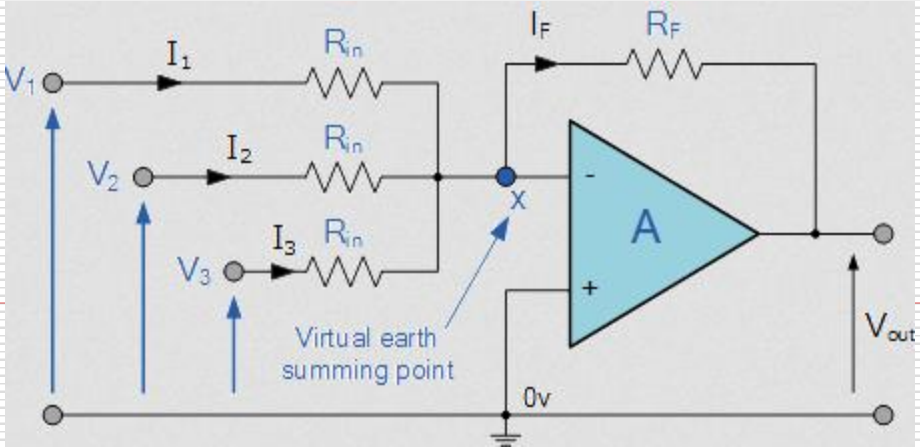
- Figure shows that the circuit is linear only over a limited range of v_i.
- When v_o exceeds about ±13V, it saturates (limits), and further increases in v_i produce no change in the output.
- The linear swing of v_o is about 4 V less than the difference in power-supply voltages.
- Although op amps usually have power-supply voltages set at ±15 V, reduced power-supply voltages may be used, with a corresponding reduction in the saturation voltages and the linear swing of v_o.



(c)

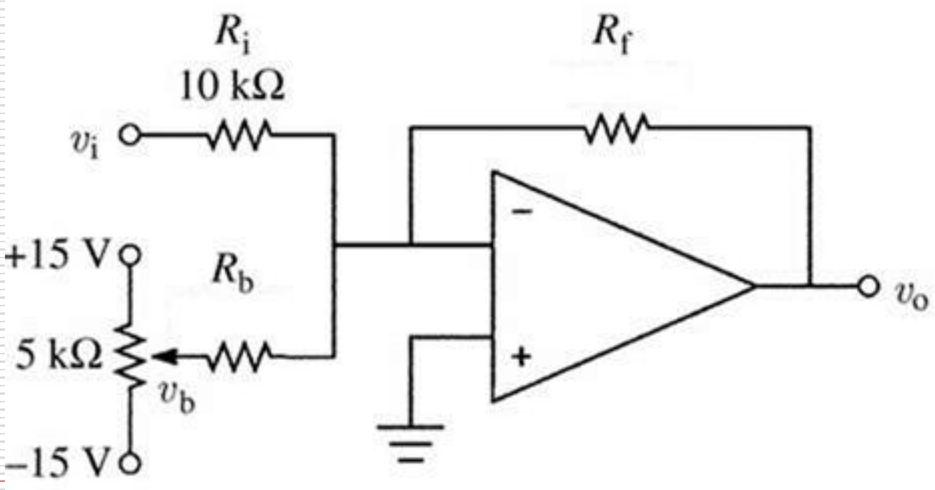
SUMMING AMPLIFIER

- The inverting amplifier may be extended to form a circuit that yields the weighted sum of several input voltages.
- Each input voltage v_{i1}, v_{i2},. . . , v_{iN} is connected to the negative input of the op amp by an individual resistor the conductance of which ($1/R_{ik}$) is proportional to the desired weighting.



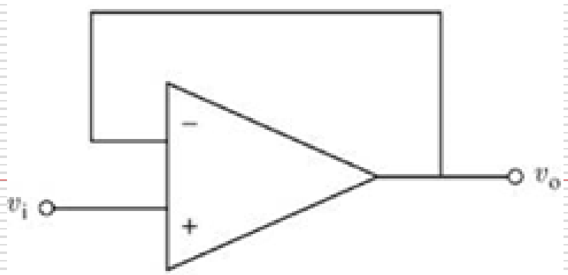
EXAMPLE

- The output of a biopotential preamplifler that measures the electro-oculogram has an undesired dc voltage of 5V at the input due to electrode half-cell potentials with a desired signal of ±1V superimposed. Design a circuit that will cancel the undesired voltage and provide a gain of -10 for the desired signal.
- Assume that potentiometer is adjusted that v_b=-10V.



NONINVERTING AMPLIFIERS: FOLLOWER

- By Rule 1, $v_i = v_o$
- The circuit is very useful as a buffer, to prevent a high source resistance from being loaded down by a low-resistance load.
- By Rule 2, no current flows into the positive input, and therefore the source resistance in the external circuit is not loaded at all.
- Input impedance is high, output impedance is low.
- When it is connected to a source with a small internal resistance, there will be no loading effect meaning no decrease in voltage at the input of op-amp.
- When another circuit is connected to it, it deliver the output voltage without any loss due its low output resistance.
- **Non-inverting amplifier** circuit that is commonly used in electronics to isolate circuits from each other especially in High-order state variable or Sallen-Key type active filters to separate one filter stage from the other.



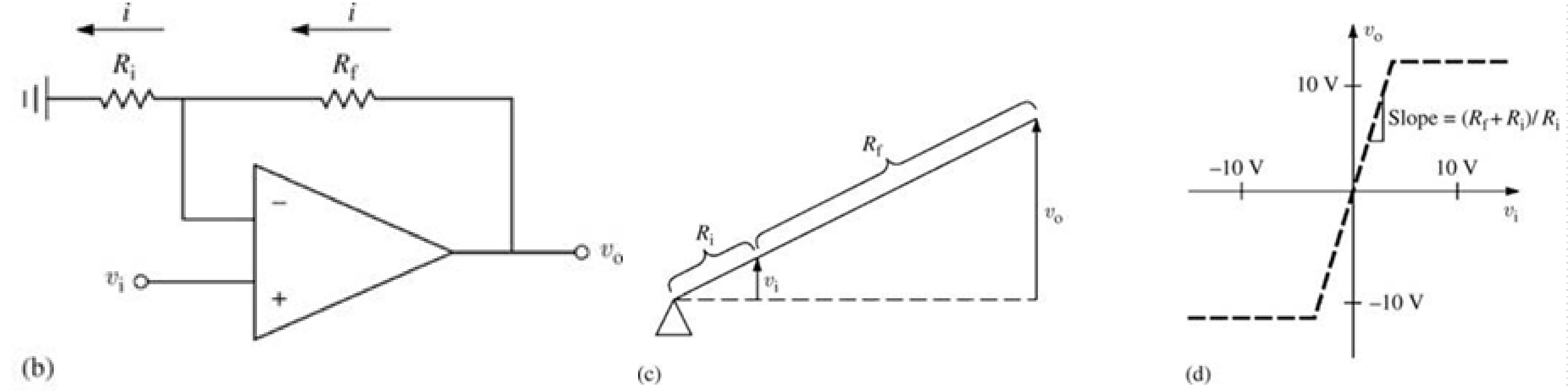
(a)

NONINVERTING AMPLIFIER

- By Rule 1, v_i appears at the negative input of the op amp.
- This causes current $i = v_i/R_i$ to flow to ground.
- By Rule 2, none of *i* can come from the negative input; therefore all must flow through Rf.
- We can then calculate $v_o = i(R_f + R_i)$ and solve for the gain.

$$\frac{v_o}{v_i} = \frac{i(R_f + R_i)}{iR_i} = \frac{R_f + R_i}{R_i}$$

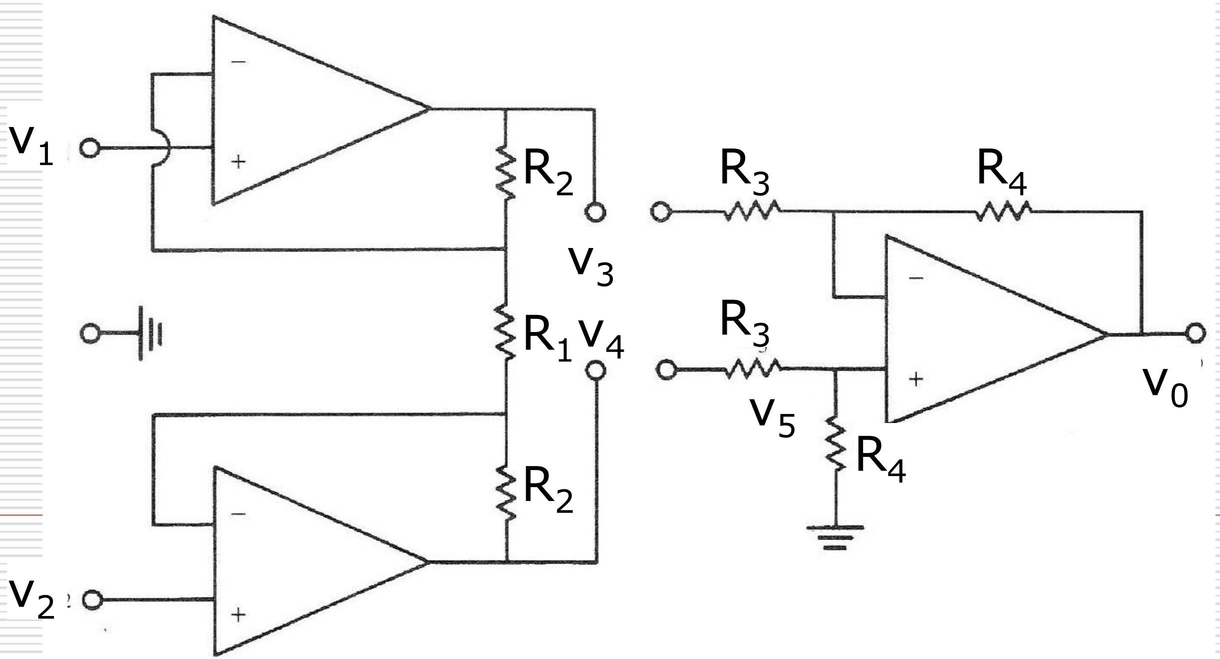
- A lever makes possible an easy visualization of the input-output characteristics. The fulcrum is placed at the left end, because R_i is grounded at the left end.
- The input-output plot shows a positive slope of (Rf + Ri)/Ri in the central portion, but the output saturates at about +13 V.



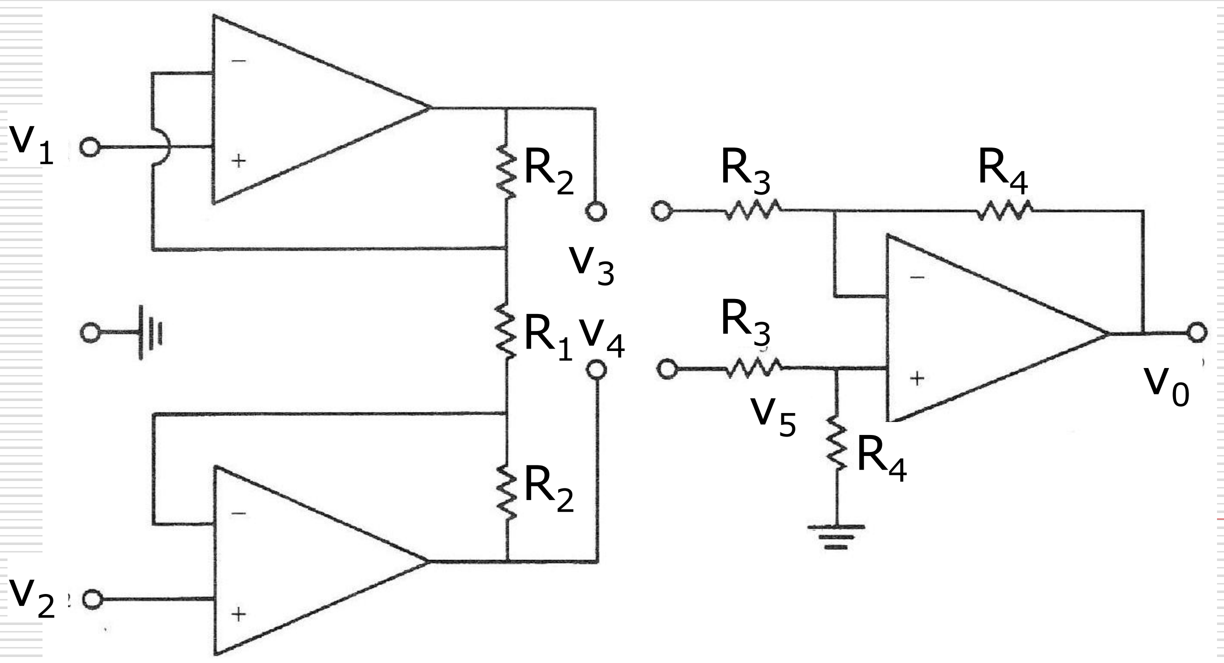
DIFFERENTIAL AMPLIFIERS

- The right side of figure shows a simple one-op-amp differential amplifier.
- By Rule 2, $v_5 = \frac{v_4 R_4}{R_3 + R_4}$, By Rule 1, $i = \frac{v_3 - v_5}{R_3} = \frac{v_5 - v_o}{R_4}$

$$v_o = \frac{(v_4 - v_3)R_4}{R_3}$$



- When $v_4 = v_3$, the differential amplifier-circuit (not op-amp) common-mode gain G_c is 0.
- When $v_4 \neq v_3$ differential gain G_d is equal to R_4/R_3
- No differential amplifier perfectly rejects the common-mode voltage. To quantify this imperfection, we use the term common-mode rejection ratio (CMRR) is defined.
- CMMR=100 for some oscilloscope differential amplifiers
- CMMR>10,000 for a high-quality biopotential amplifier.

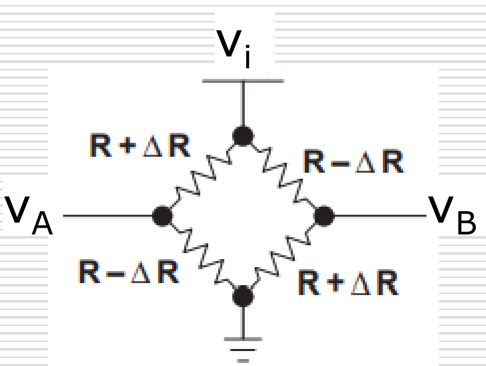


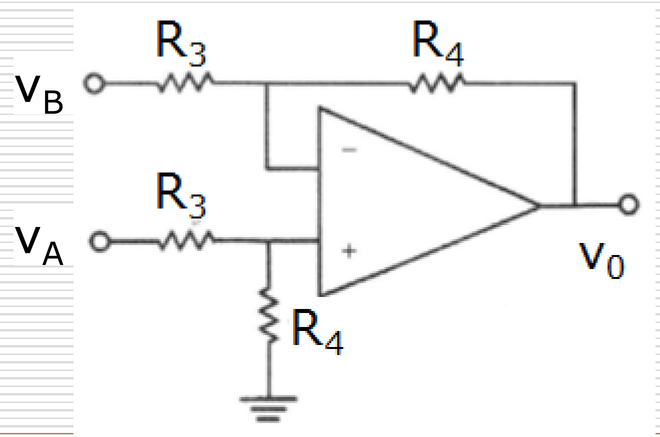
$$v_o = \frac{(v_4 - v_3)R_4}{R_3}$$

$$\text{CMRR} = \frac{G_d}{G_c}$$

EXAMPLE

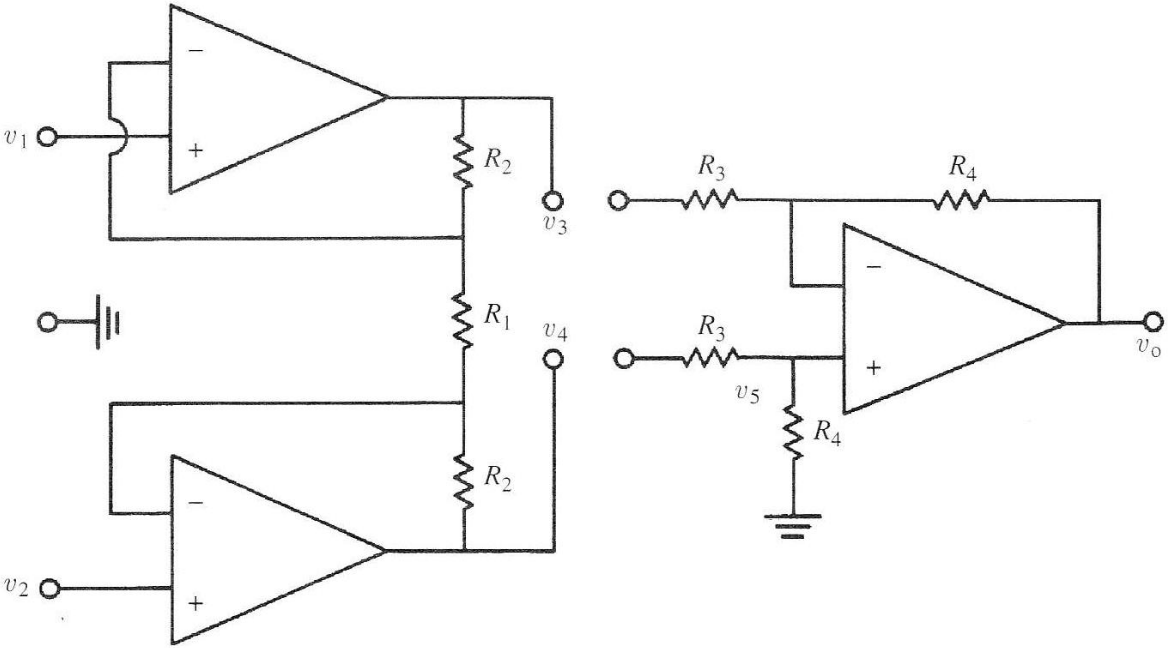
- A blood-pressure sensor uses a four-active-arm Wheatstone strain gage bridge excited with 5 V dc. Arm resistance is 1kΩ.
- At full scale, each arm changes resistance by +0.1%.
- Calculate the resistance R4, if R3 is equal to 1kΩ so that differential amplifier can provide a full-scale output over the op-amp's full range of linear operation (5V).





THREE-OP-AMP DIFFERENTIAL AMPLIFIER

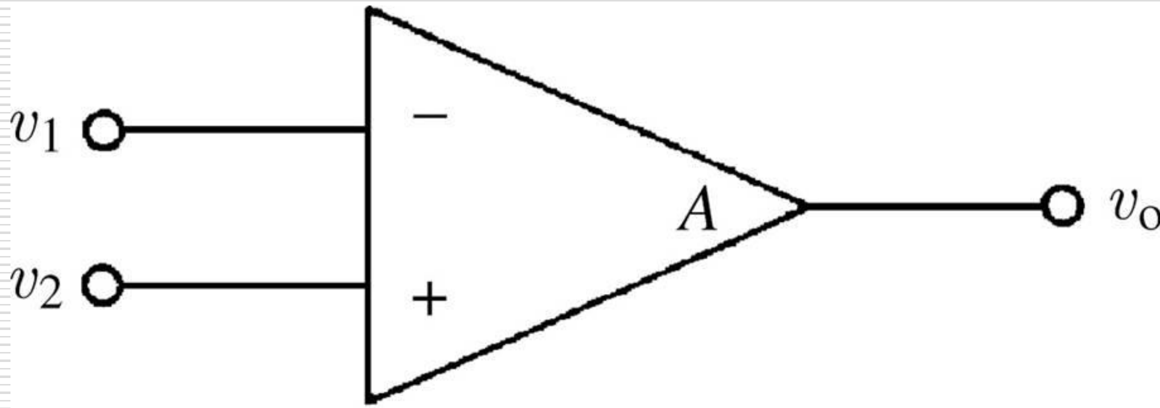
- Right half of the circuit has low input impedance.
- Adding two followers separately to inputs can increase the input impedance but does not affect CMMR.
- Using circuit on the left both CMMR and input impedance are increased.
- The resulting three-op-amp amplifler circuit is frequently called an instrumentation amplifier.
- This circuit finds wide use in measuring biopotentials because it rejects the large 60 Hz common-mode voltage that exists on the body.

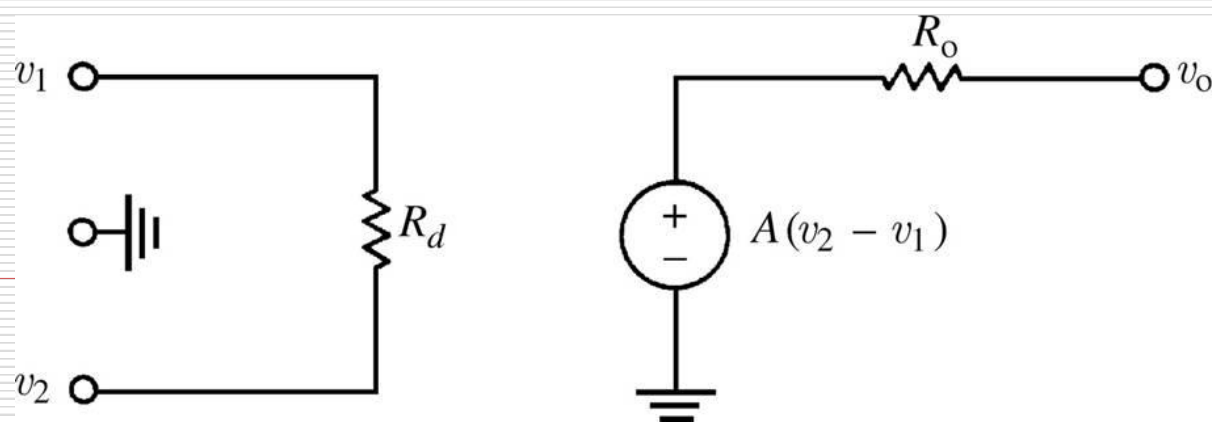


$$G_d = \frac{v_3 - v_4}{v_1 - v_2} = \frac{2R_2 + R_1}{R_1}$$

Comparator

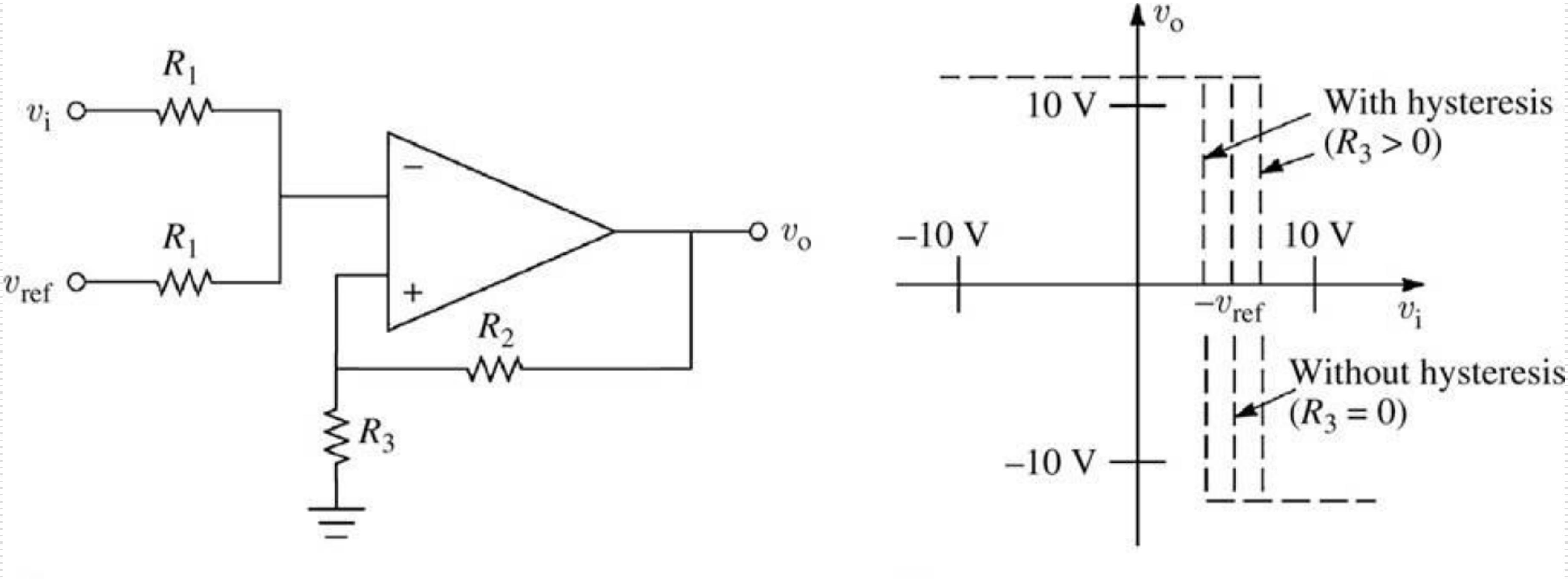
- Compares the input voltage with some reference voltage.
- Op amp goes into positive or negative saturation depending on the inputs.
- If $v_{ref}=v_2>v_1$ then $v_o=13V$, otherwise $v_o=-13$.





Positive Feedback Comparator

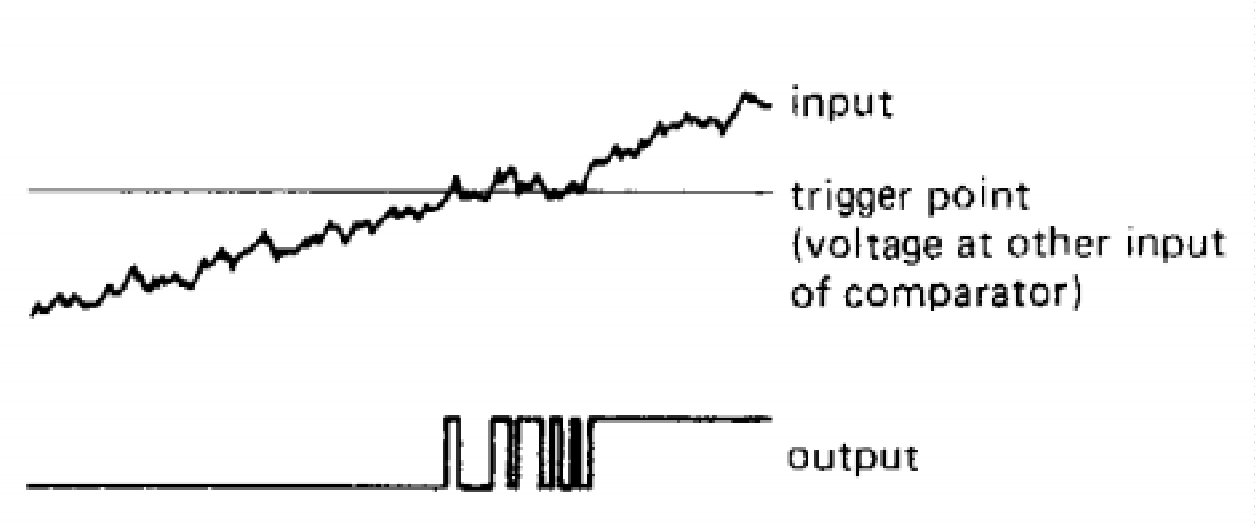
- The input circuit may be expanded by adding the two R1 resistors shown in Figure (a).
- This provides a known input resistance for the circuit and minimizes overdriving the op-amp input.
- Figure (b) shows that the comparator flips when v_i=-v_{ref} , R_3=0.



(a) (b)

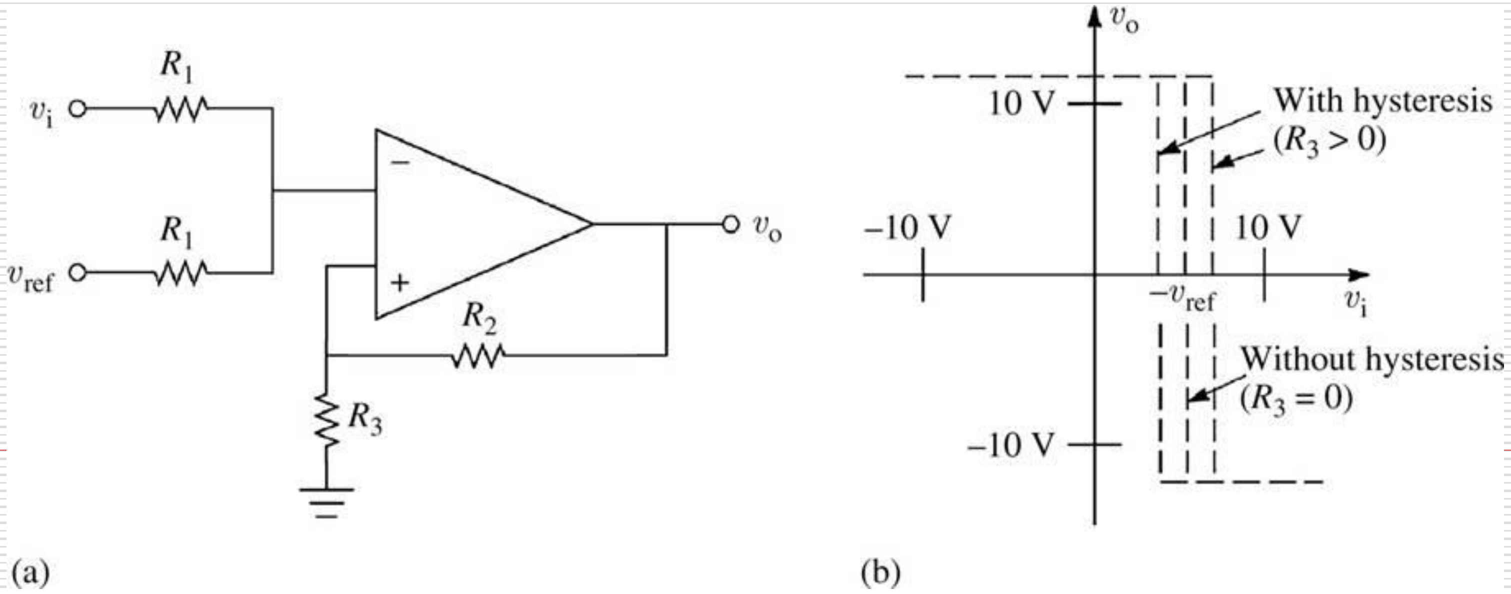
Disadvantage of Simple Comparator

- If the input is noisy, output take several transitions as input passes the trigger point



Positive Feedback Comparator

- Example: Vi = 10 V, Vref = -5 V -> V- = +2.5 V -> Vo = -13 V; -> V+ = -1 V (by selecting as R2=12xR3)
- Vi needs to be slightly less 3 V, then V- <-1V -> Vo = +13, -> V+=+1 V
- Vi needs to be slightly more than 7 V, for Vo = -13 V.
- Width of hystresis, 4V is four times bigger than voltage accross R3.
- Width of the hystresis can be adjusted using a potentiometer as R3.
- Noise can not fluctuate output anymore.

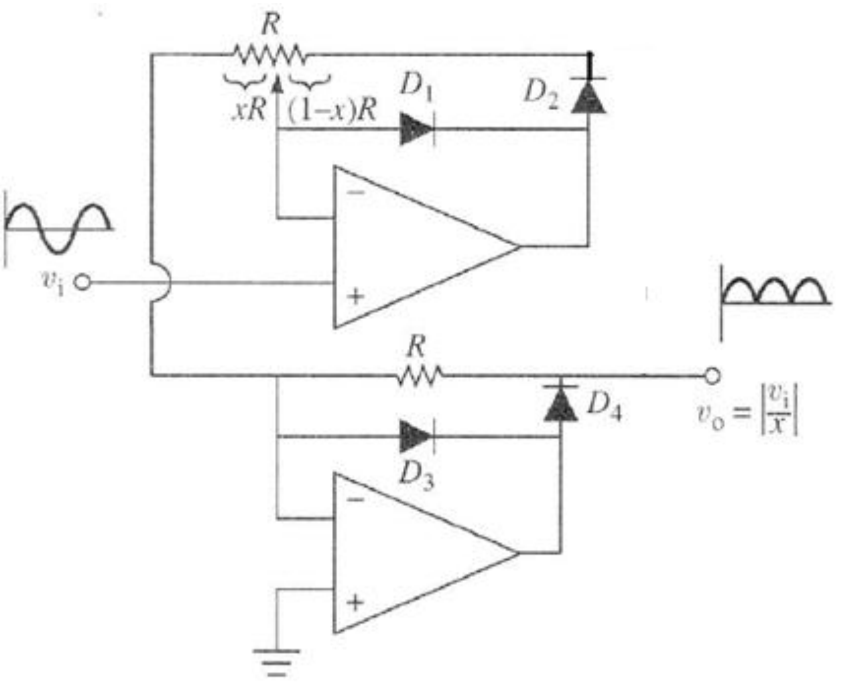


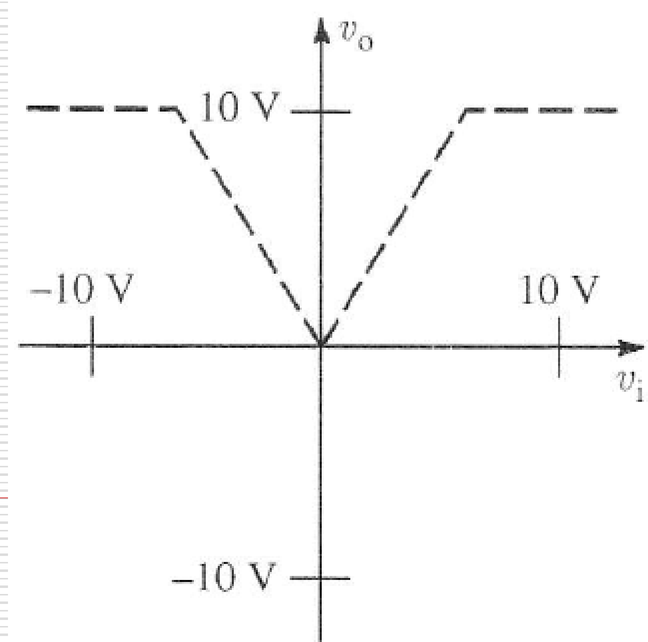
Full-wave precision rectifier

- A common rectifying circuit you build for a power supply will work perfectly, but it will be completely useless for high-precision signal processing circuits. The reason is simply that in many applications the signal we would like to rectify will be less than the voltage needed to turn a diode on. Even small-signal germanium diodes require about 0.3 volts to turn on.
- The precision rectifier, also known as a super diode, is a configuration obtained with one or more operational amplifiers in order to have a circuit behave like an ideal diode and rectifier.
- Simple resistor-diode rectifiers do not work well for voltages below 0.7 V, because the voltage is not sufflcient to overcome the forward voltage drop of the diode. This problem can be overcome by placing the diode within the feedback loop of an op amp, thus reducing the voltage limitation by a factor equal to the gain of the op amp.

Full-wave precision rectifier

- For $v_i > 0$, D2 and D3 ON, D1 and D4 OFF. Upper op amp circuit becomes a noninverting amplifier with gain of 1/x , lower op amp circuit has no effect on output
- For $v_i < 0$, D2 and D3 OFF, D1 and D4 ON. Lower op amp circuit becomes an inverting amplifier with gain of -1/x.
- Circuit gain may be adjusted with a single pot.





LOGARITHMIC AMPLIFIERS

- Makes use of the nonlinear volt–ampere relation of the silicon planar transistor

$$V_{BE} = 0.060 \log\left(\frac{I_C}{I_S}\right)$$

V_{BE} = base – emitter voltage

I_C = collector current

I_S = reverse saturation current, 10^{-13} A at 27°C

- These log and antilog circuits are used to
  - multiply a variable, divide it, or raise it to a power;
  - to compress large dynamic ranges into small ones;
  - to linearize the output of devices with logarithmic or exponential input-output relations.

Log Rules:

1) $\log_b(mn) = \log_b(m) + \log_b(n)$

2) $\log_b(m/n) = \log_b(m) - \log_b(n)$

3) $\log_b(m^n) = n \cdot \log_b(m)$

- In photometer the logarithmic converter can be used to convert transmittance to absorbance.

- Output $v_o = V_{BE}$ is logarithmically related to v_i as given by (3.8) over the approximate range 10^{-7} A < I_C < 10^{-2} A.
- The approximate range of v_o is -0.36 to -0.66 V, so larger ranges of v_o are sometimes obtained by the alternate switch the circuit gain is increased by 10.

Log Rules:

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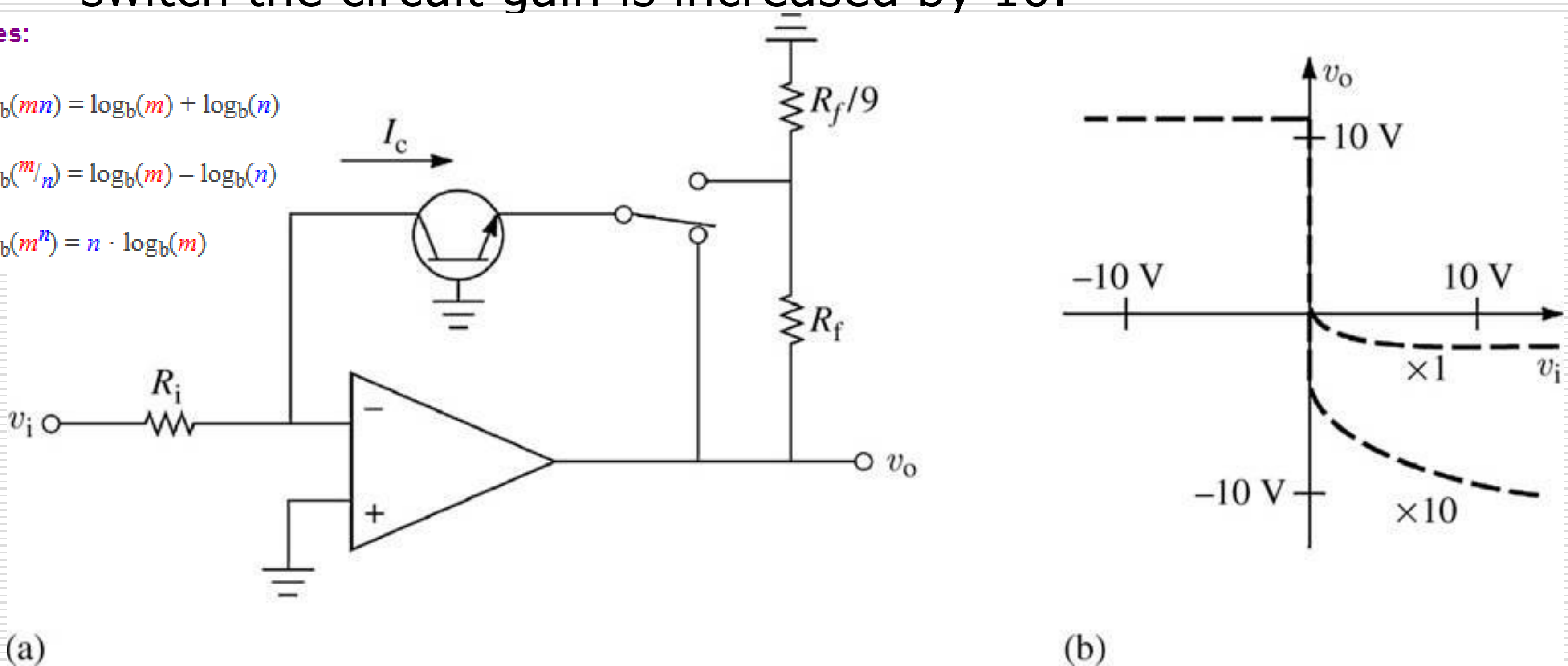


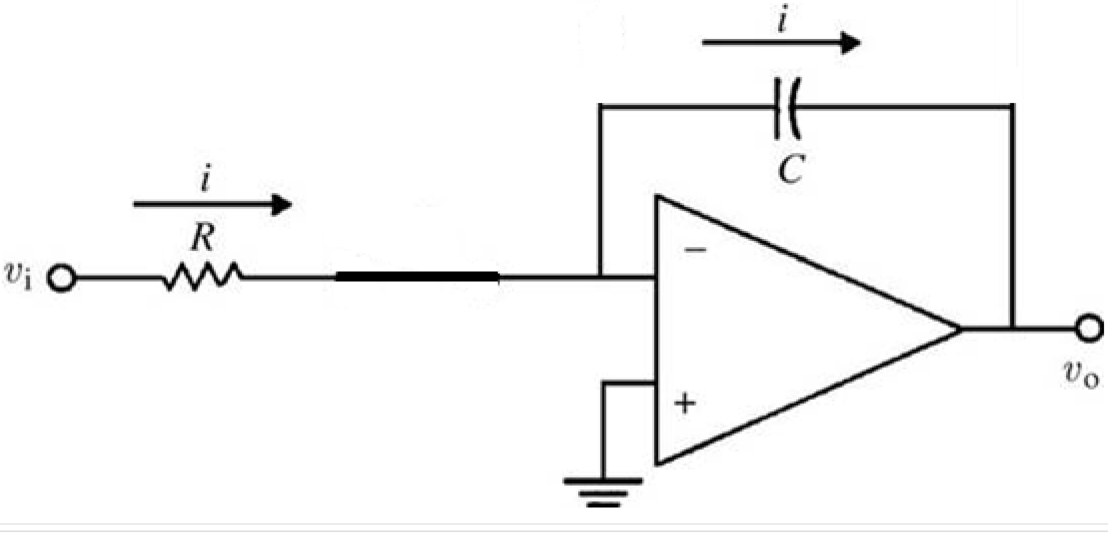
Integrator

- Voltage across an uncharged capacitor $v = \frac{1}{C}\int_0^{t_1} i dt$

  where i is the current through C and t_1 is the integration time.

- For the integrator, for v_i positive, the input current $i = v_i/R$ flows through C in a direction to cause v_o to move in a negative direction.

$$v_o = -\frac{1}{RC}\int_0^{t_1} v_i dt + v_{ic}$$

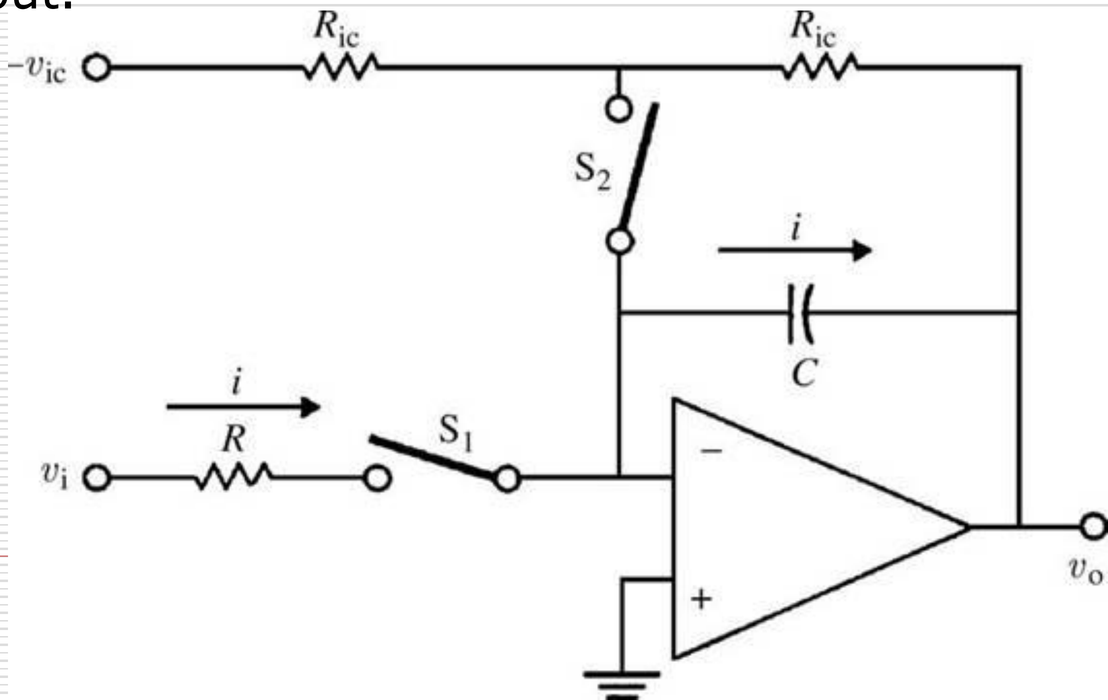


A Three-Mode Integrator

- With S1 open and S2 closed, the dc circuit behaves as an inverting amplifier.
- Thus $v_o = vi_c$ and vo can be set to any desired initial condition.
- With Si closed and S2 open, the circuit integrates.
- With both switches open, the circuit holds v_o constant, making possible a leisurely readout.

$$v = \frac{1}{C}\int_0^{t_1} i\,dt$$

$$v_o = -\frac{1}{RC}\int_0^{t_1} v_i\,dt + v_{ic}$$



A Three-Mode Integrator

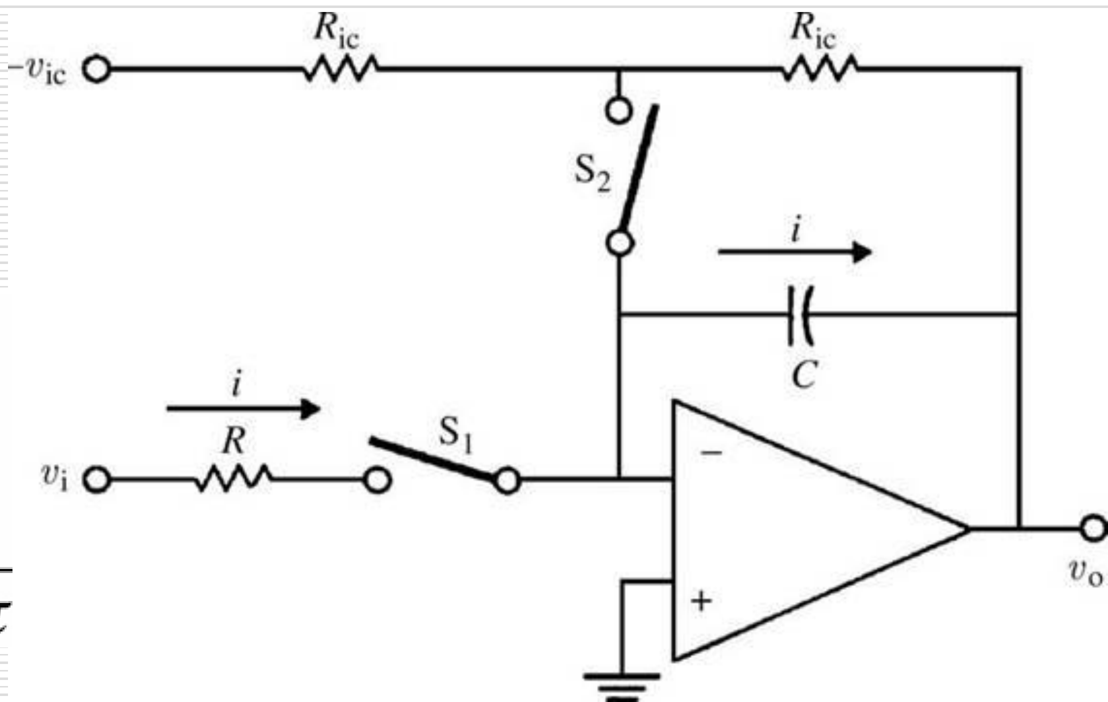
- The frequency response of an integrator is easily analyzed because the formula for the inverting amplifier gain can be generalized to any input and feedback impedances. Thus for Figure below , with S1 closed,

$$v = \frac{1}{C}\int_0^{t_1} i\,dt$$

$$v_o = -\frac{1}{RC}\int_0^{t_1} v_i\,dt + v_{ic}$$

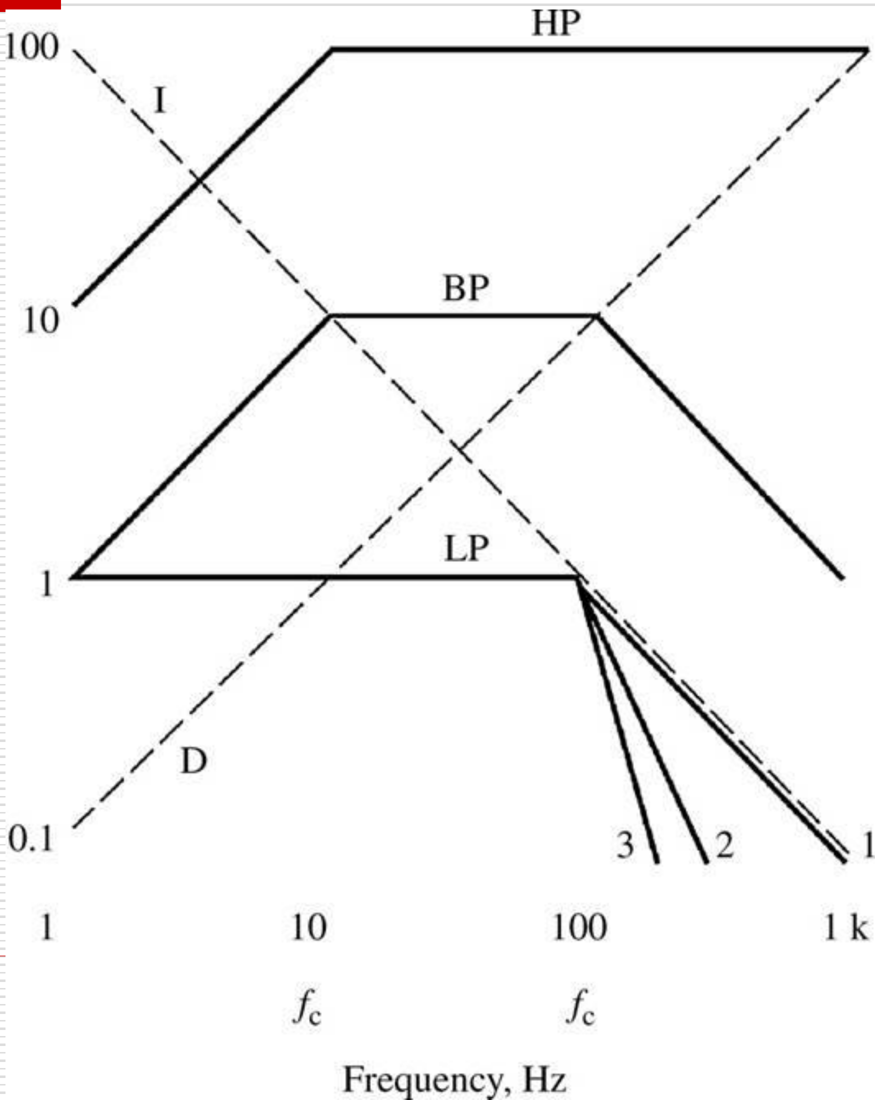
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_f}{Z_i} = -\frac{1/j\omega C}{R}$$

$$= -\frac{1}{j\omega RC} = -\frac{1}{j\omega\tau}$$



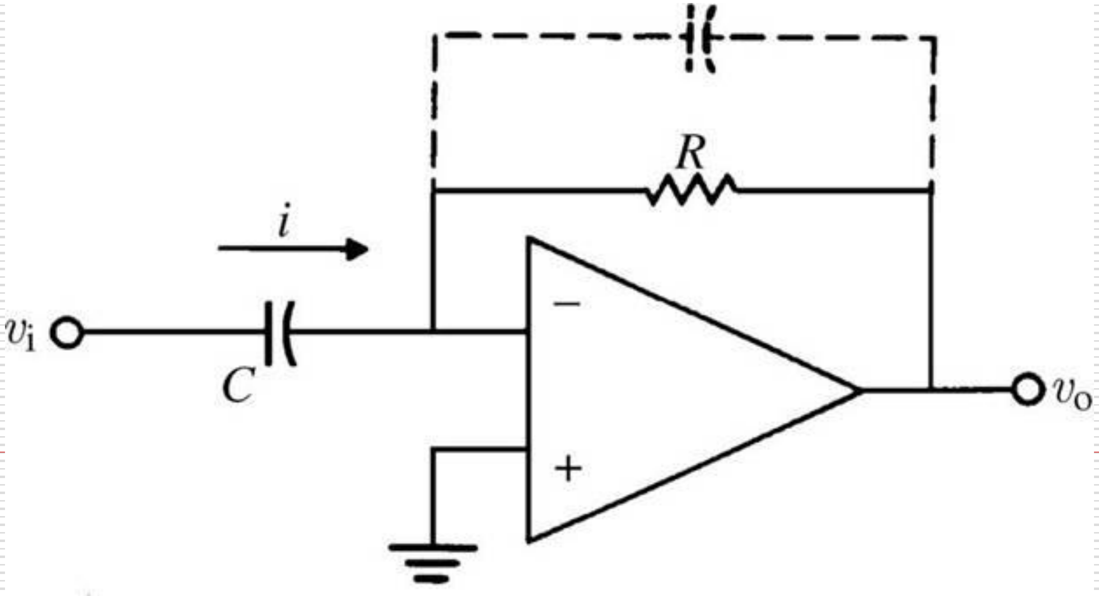
Bode plot (gain versus frequency) for various filters.

- Integrator (I);
- differentiator (D);
- low pass (LP), 1, 2, 3 section (pole);
- high pass (HP);
- bandpass (BP).
- Corner frequencies *fc* for high-pass, low-pass, and bandpass filters.



Differentiator

- Mathematical operation - Differentiation of signal
- Interchanging the integrator's *R* and *C* in earlier circuit yields the differentiator
- A differentiator followed by a comparator is useful for detecting an event the slope of which exceeds a given value—for example, detection of the R wave in an electrocardiogram.



- Current through a capacitor is given by

$$i = C\frac{dv}{dt}$$

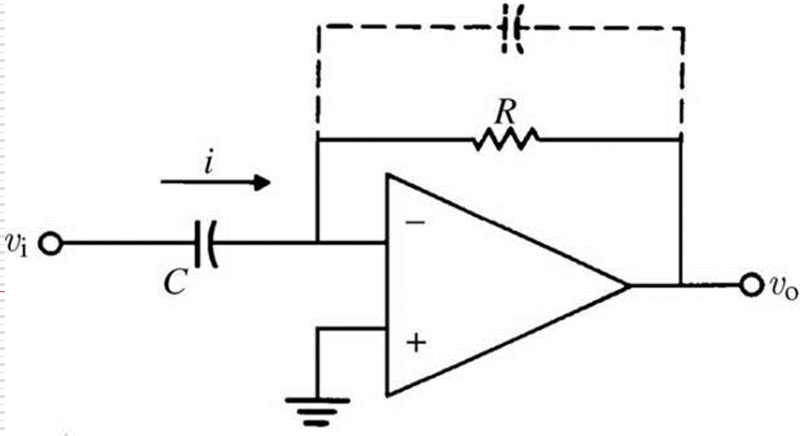
- If dv_i/dt is positive, i flows through R in a direction such that it yields a negative v0.

$$v_o = -RC\frac{dv_i}{dt}$$

- The frequency response of a differentiator is given by the ratio of feedback to input impedance.

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R}{1/j\omega C} = -j\omega RC = -j\omega\tau$$

- Circuit gain increases as f increases and that it is equal to unity when $w\tau = 1$.



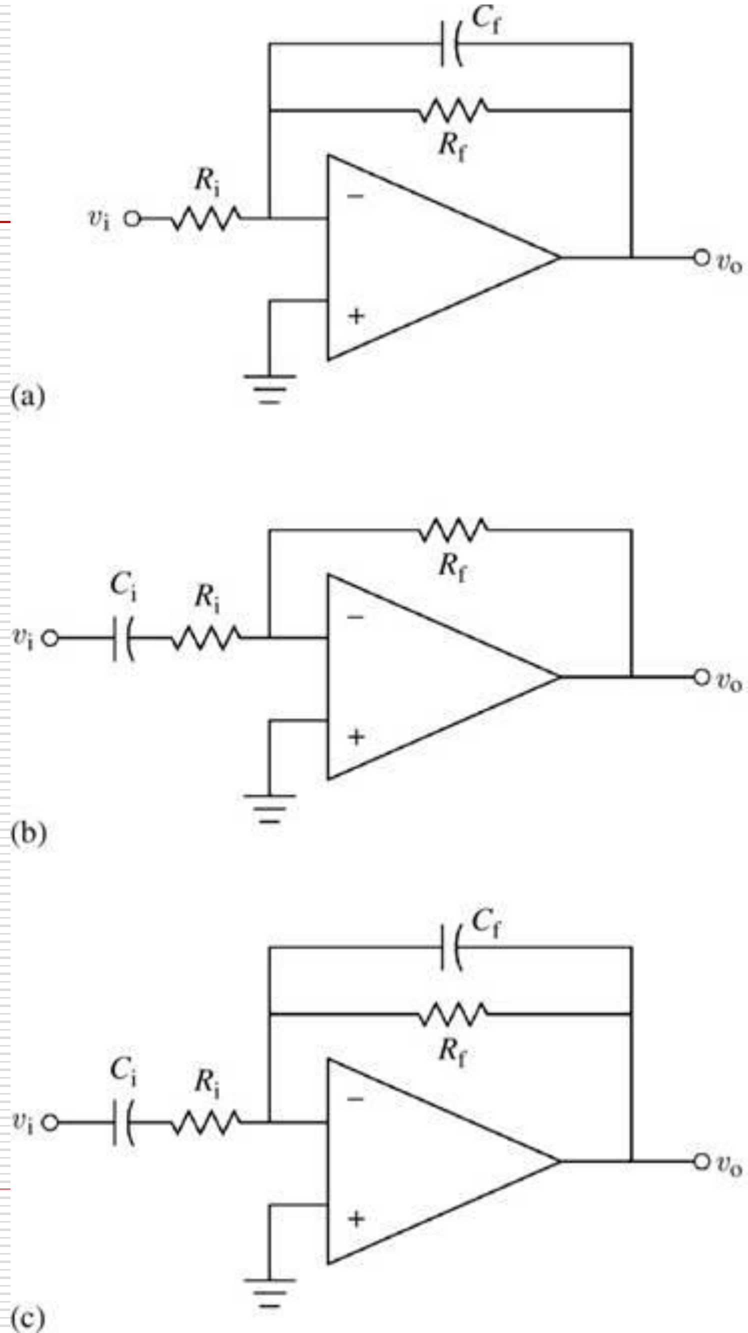
- Active filters. (a) A low-pass filter attenuates high frequencies, (b) A high-pass filter attenuates low frequencies and blocks dc. (c) A bandpass filter attenuates both low and high frequencies.

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{\dfrac{(R_f/j\omega C_f)}{[(1/j\omega C_f)+R_f]}}{R_i}$$

$$= \frac{R_f}{(1+j\omega R_f C_f)R_i} = -\frac{R_f}{R_i}\frac{1}{1+j\omega\tau}$$

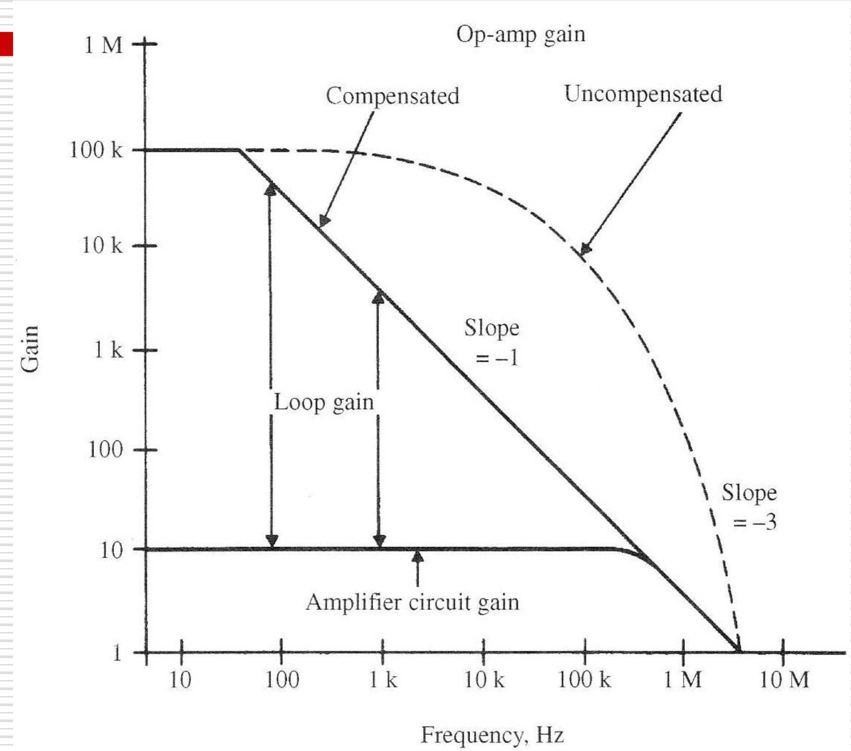
$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R_f}{1/j\omega C_i + R_i}$$

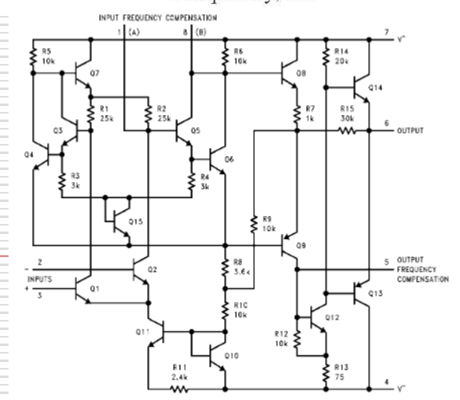
$$= -\frac{j\omega R_f C_i}{1+j\omega C_i R_i} = -\frac{R_f}{R_i}\frac{j\omega\tau}{1+j\omega\tau}$$



FREQUENCY RESPONSE

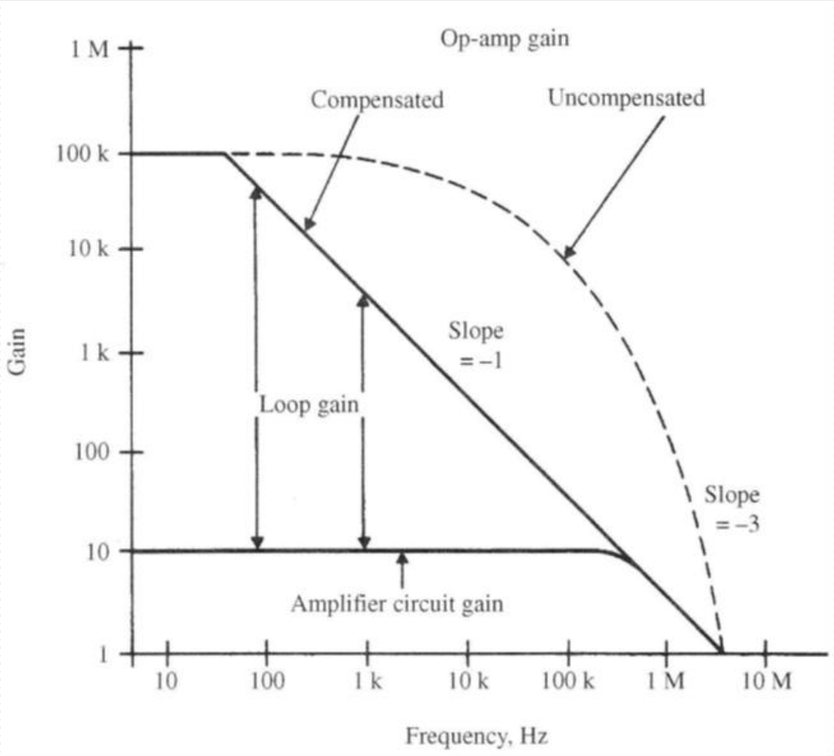
- Up until now, we have found it useful to consider the op amp as ideal. Now we shall examine the effects of several nonideal characteristics, starting with that of frequency response.
- In order to produce high gain, op amp has several stages each of which has capacitance behaving like a simple RC low-pass filter reduces high-frequency gain.
- Some opamps needs compesantion by adding an external capacitor to have -1 slope and maximum -90° phase shift.
- For an amplifier, if the gain is greater than 1 when the phase shift is equal to -180° (the closed-loop condition for oscillation), there is undesirable oscillation.





Gain-Bandwidth Product

- Gain-bandwidth product of the op amp is equal to the product of gain and bandwidth at a particular frequency.
- Unity-gain-bandwidth product is 4 MHz, a typical value for op amps.
- Along the entire curve with a slope of -1, the gain-bandwidth product is still constant, at 4 MHz.
- Higher-frequency applications, op amps such as the OP-37E are available with gain-bandwidth products of 60 MHz.



What is dB and cut-off frequecy

- The decibel (symbol: dB) is a logarithmic unit used to express the ratio of one value of a physical property to another, and may be used to express a change in value (e.g., +1 dB or -1 dB) or an absolute value.
- Suppose a signal has a power of P1 watts, and a second signal has a power of P2 watts. Then the power amplitude difference in decibels, symbolized SdBP, is:

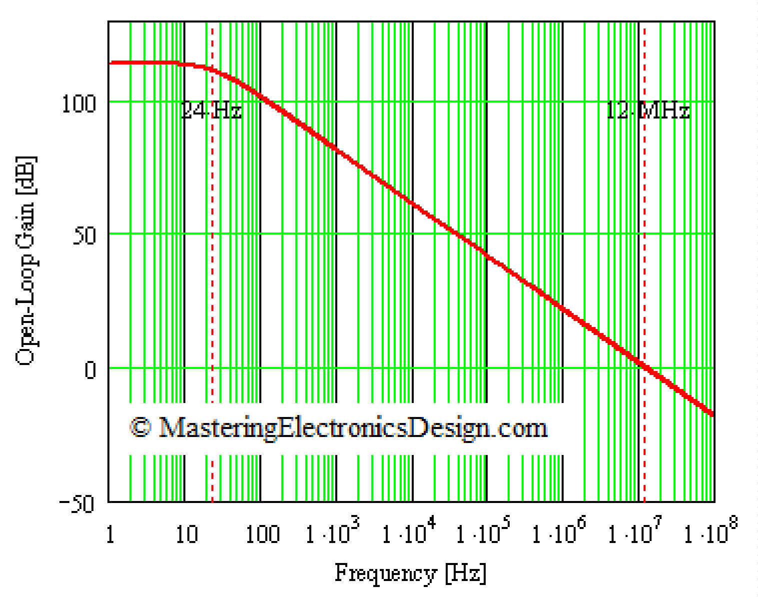
  SdBP = 10 log10 (P2 / P1)
- If SdBP expresses the ratio of a value to a reference value; such as 1 volt, then the suffix is "V" (i.e., "20 dBV"), and if the reference value is one milliwatt, then the suffix is "m" (i.e., "20 dBm") is added.
- Cutoff frequency or corner frequency is the frequency at which the power output of a circuit, such as a line, amplifier, or electronic filter has fallen to the half of the power in the passband.

$$10\log\left(\frac{A/2}{A}\right) = -3dB$$

Frequency vs Open Loop Gain of an Op-Amp

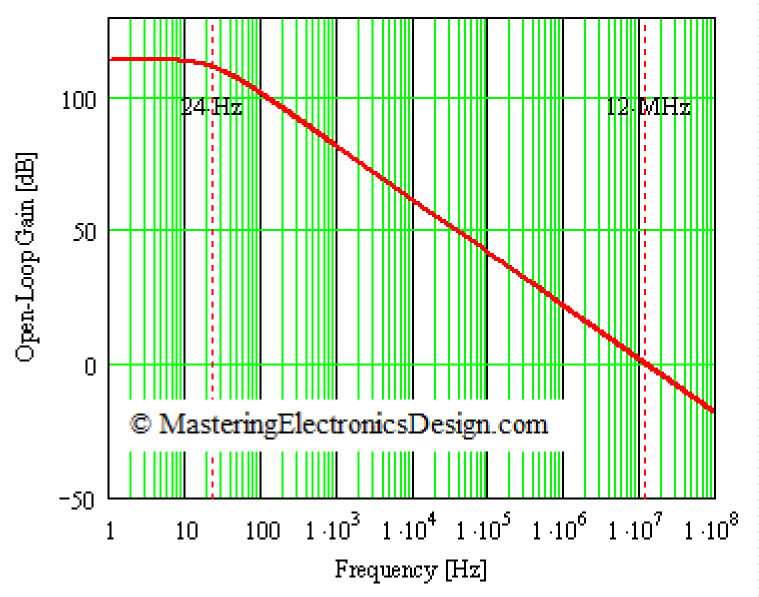
- In the case of ADA4004, the gain bandwidth product is 12 MHz. This means that, at a gain of one, the bandwidth is 12 MHz.
- At the maximum open-loop gain of 500000, the bandwidth is 12 MHz divided by 500000, which is 24 Hz. This is the op amp open-loop cutoff frequency.
- For simplicity and clarity, the gain is always shown in dB.
- One can see that, at low frequencies, the gain is 20 log(500000) = 114 dB.
- Why is the gain 111 dB at 24 Hz? Because the cutoff frequency is at -3 dB, which means that the corresponding gain is 114dB – 3dB = 111dB.
- Starting with the cutoff frequency of 24 Hz, the gain rolls off at a rate of 20dB/decade until 12 MHz, where the gain is 1, or 0 dB.

$$Ao(\omega) = \frac{Aol}{1 + \frac{j \cdot \omega}{2 \cdot \pi \cdot fc}}$$



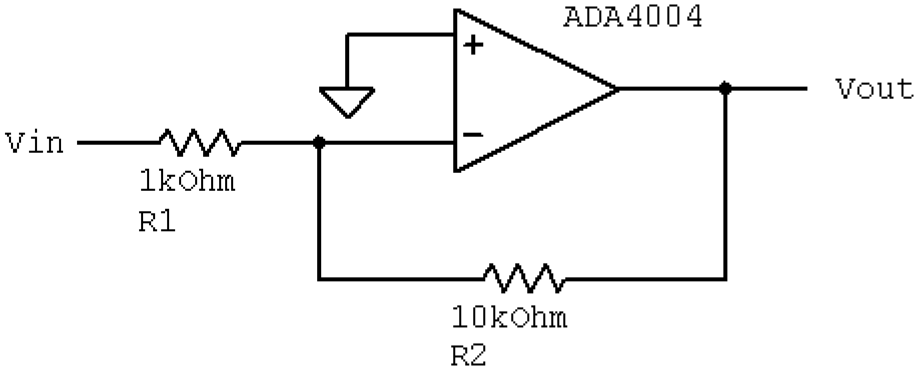
Closed-Loop Gain

- It might appear that the op amp has very poor frequency response, because its gain is reduced for frequencies above 24 Hz.
- However, an amplifier circuit is never built using the op-amp open loop, so we shall therefore discuss only the circuit closed-loop response.
- Gain of op-map circuit using closed loop is much lower than the open loop gain in return greatly extended frequency response.
- Gain can be calculated using gain-bandwidth product.



Example

- As an example, Figure 2 shows an inverting amplifier with ADA4004.
- The gain is set by the ratio between R2 and R1
- The resistor ratio is 10, so the bandwidth is 12 MHz divided by 10 which is 1.2 MHz.

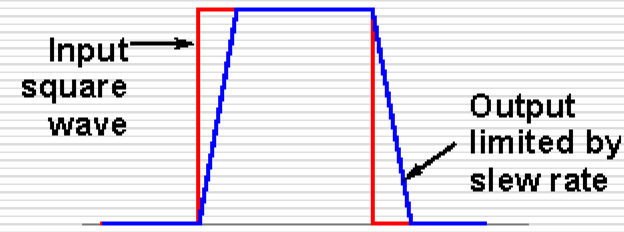


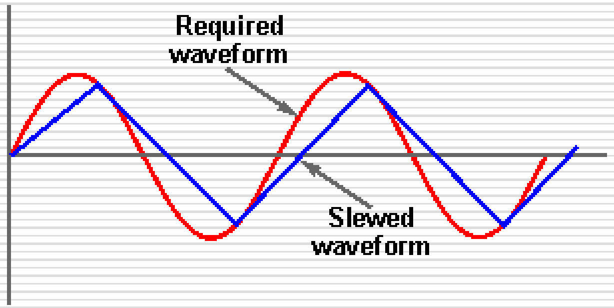
SLEW RATE

- The slew rate of an op amp or any amplifier circuit is the rate of change in the output voltage caused by a step change on the input.
- If an op amp is operated above its slew rate limit, signals will become distorted. The easiest way to see this is to look at the example of a sine wave.

$$Slew\ Rate = 2 . \pi . f . V$$

  - Slew rate is measured in volts / second, although actual measurements are often given in v/µs
  - f = the highest signal frequency, Hz
  - V = the maximum peak voltage of the signal.
- As an example, take the scenario where an op amp is required to amplify a signal with a peak amplitude of 5 volts at a frequency of 25kHz. An op amp with a slew rate of at least 2 π x 25 000 x 5 = 0.785V/µs would be required.



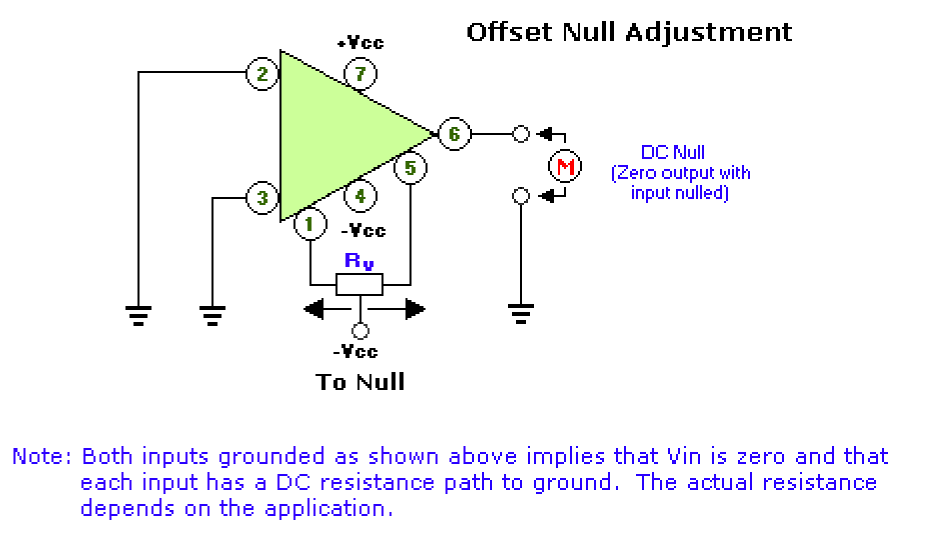


Offset Voltage

- The two op-amp inputs drive the bases of transistors, and the base-to-emitter voltage drop may be slightly different for each.
- To obtain vo = 0, the voltage (v1 - v2) must be a few millivolts.
- Offset voltage is usually not important when vi is 1-10 V.
- Problem, when vi is on the order of millivolts,
- Example: Amplifying the output from thermocouples or strain gages, the offset voltage must be considered.

Nulling

- Offset voltage may be reduced to zero by adding an external nulling pot to the terminals indicated on the specification sheet.
- Adjustment of this pot increases emitter current through one of the input transistors lowers it through the other.
- Alters the base-to-emitter voltage of the two transistors until the offset voltage is reduced to zero.

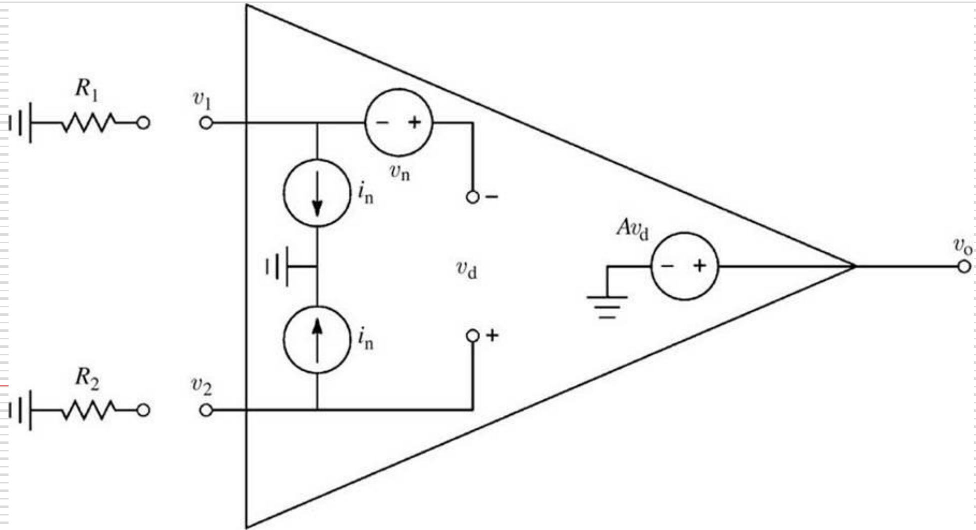


DRIFT

- Even though the offset voltage may be set to 0 at 25°C, it does not remain there if temperature is not constant.
- Temperature changes that affect the base-to-emitter voltages may be due to either environmental changes or to variations in the dissipation of power in the chip that result from fluctuating output voltage.
- The effects of temperature may be specified as a maximal offset voltage change in volts per degree Celsius or a maximal offset voltage change over a given temperature range, say -25°C to +85° C.
- If the drift of an inexpensive op amp is too high for a given application, tighter specifications (0.1μV/°C) are available with temperature-controlled chips.

Noise

- All semiconductor junctions generate noise, which limits the detection of small signals.
- Op amps have transistor input junctions, which generate both noise-voltage sources and noise-current sources.
- For low source impedances R1 and R2, only the noise voltage v_n is important; it is large compared with the i_nR drop caused by the current noise i_n.
- The noise is random, but the amplitude varies with frequency.
- For example, at low frequencies the noise power density varies as 1/f (flicker noise), so a large amount of noise is present at low frequencies.
- At the midfrequencies, the noise is lower and can be specified in rms units of $V.Hz^{-1/2}$.

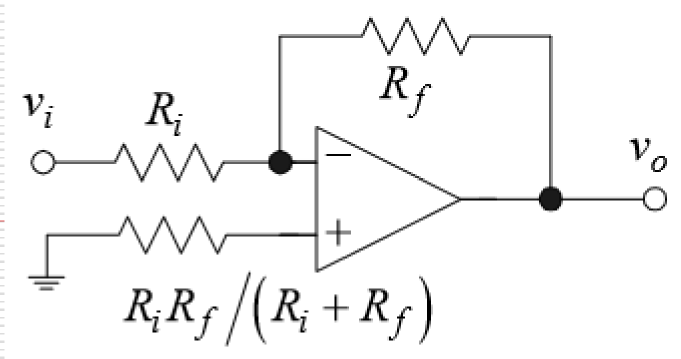


Bias Current

- Because the two op-amp inputs drive transistors, base or gate current must flow all the time to keep the transistors turned on.
- This is called bias current, which for the 411 is about 200 pA.
- This bias current must flow through the feedback network.
- It causes errors proportional to feedback-element resistances.
- To minimize these errors, small feedback resistors, such as those with resistances of 10 kΩ, are normally used.
- Smaller values should be used only after a check to determine that the current flowing through the feedback resistor, plus the current flowing through all load resistors, does not exceed the op-amp output current rating (20 mA for the 411).

Differential Bias Current

- The difference between the two input bias currents is much smaller than either of the bias currents alone.
- A degree of cancellation of the effects of bias current can be achieved by having each bias current flow through the same equivalent resistance.
- This is accomplished for the inverting amplifier and the noninverting amplifier by adding, in series with the positive input, a compensation resistor the value of which is equal to the parallel combination of Ri and Rf.
- There still is an error, but it is now determined by the difference in bias current.



Drift

- The input bias currents are transistor base or gate currents,
- They are temperature-sensitive,
  - because transistor gain varies with temperature.
- The changes in gain of the two transistors tend to track together,
  - so the additional compensation resistor that we have described minimizes the problem.

- Figure shows how variations in bias current contribute to overall noise. The noise currents flow through the external equivalent resistances so that the total rms noise voltage is

$$v \cong \{[v_n^2 + (i_n R_1)^2 + (i_n R_2)^2 + 4\kappa T R_1 + 4\kappa T R_2]\mathrm{BW}\}^{1/2}$$

- The specification sheet provides values of v_n and i_n (sometimes v_{n2} and i_{n2}), thus making it possible to compare different op amps.
- For small (10 kΩ) source resistances, BJT input op amp produces smaller noise
- For larger source resistances, FET input op amp produces smaller noise

R_1 and R_2 = equivalent source resistances

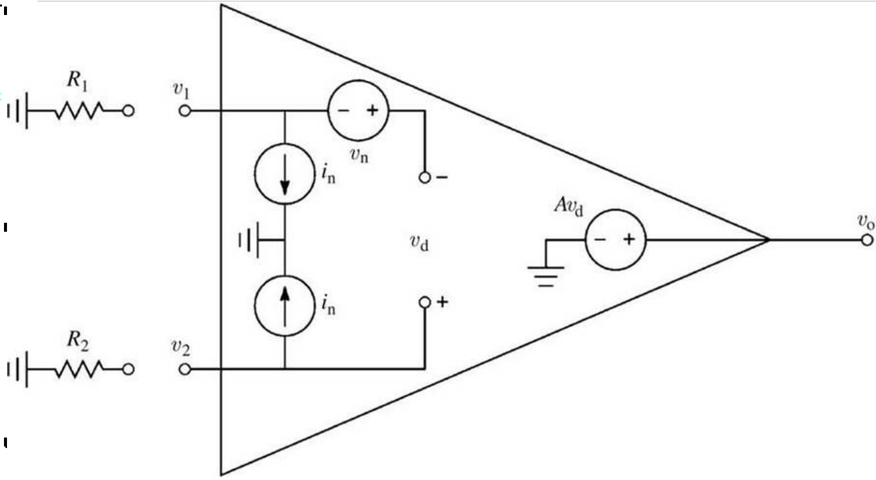
v_n = mean value of the rms noise voltage, in $\mathrm{V \cdot Hz^{-1/2}}$, across the frequency range of interest

i_n = mean value of the rms noise current, in $\mathrm{A \cdot Hz^{-1/2}}$, across the frequency range of interest

κ = Boltzmann's constant (Appendix)

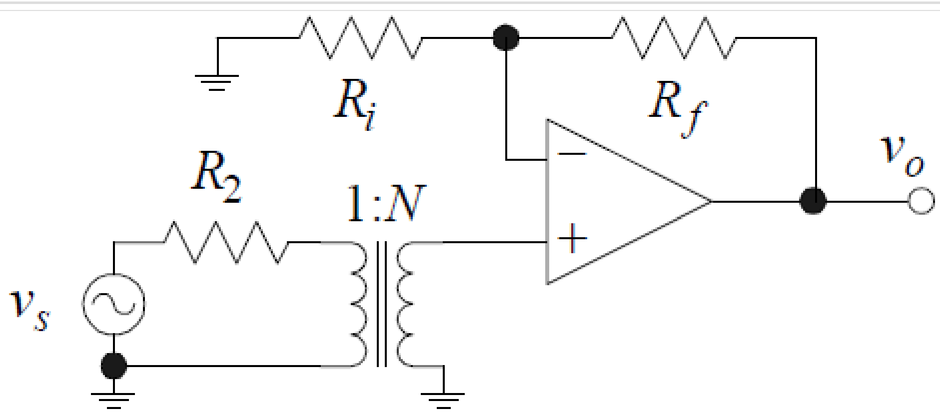
T = temperature, K

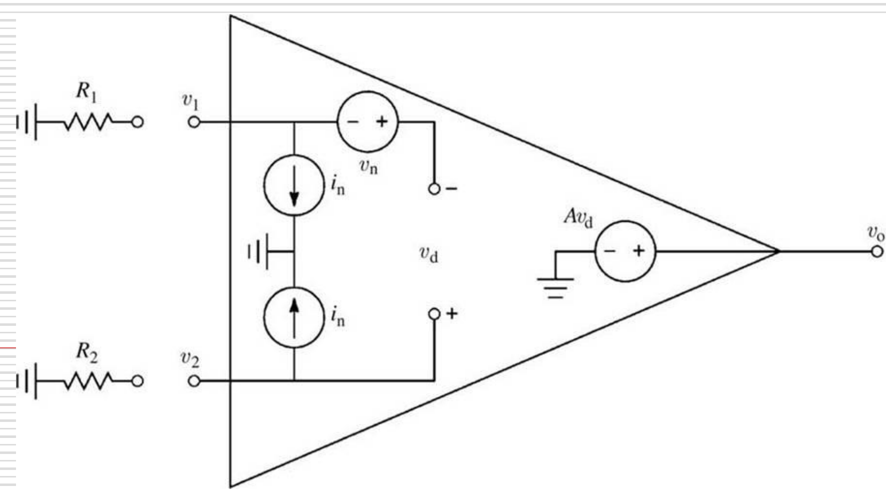
BW = noise bandwidth, Hz



$$v \cong \{[v_n^2 + (i_n R_1)^2 + (i_n R_2)^2 + 4\kappa T R_1 + 4\kappa T R_2]\mathrm{BW}\}^{1/2}$$

- Low noise ac amplifier design (noninverting amplifier) by impedance matching
- Characteristic noise resistance is $R_n = v_n / i_n$
- Set $R_n = R_2$ using a transformer with turns ratio 1:N where $N = (R_n/R_2)^{1/2}$





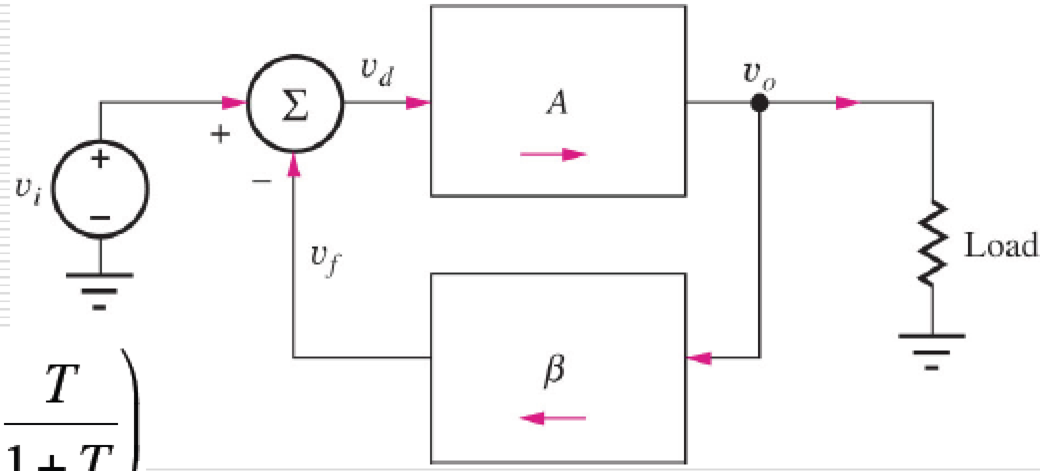
Classic Feedback Systems

- A = transfer function of open-loop amplifier or open-loop gain.
- β = transfer function of feedback network.
- T = loop gain = Aβ
  - For negative feedback: T(s) > 0
  - For positive feedback: T(s) < 0

$$v_d = v_i - v_f$$

$$v_o = A v_d$$

$$v_f = \beta v_o$$



$$A_v = \frac{v_o}{v_i} = \frac{A}{1 + A\beta} = \frac{1}{\beta}\left(\frac{A\beta}{1 + A\beta}\right) = A_v^{Ideal}\left(\frac{T}{1+T}\right)$$

Gain Error and Fractional Gain Error

- Gain Error: GE = (ideal gain) - (actual gain)
- For non-inverting amplifier,

$$GE = \frac{1}{\beta} - \frac{1}{\beta}\left(\frac{T}{1+T}\right) = \frac{1}{\beta}\left(\frac{1}{1+T}\right)$$

- Fractional or percentage error:

$$FGE = \frac{GE}{A_v^{Ideal}} = \frac{\frac{1}{\beta}\left(\frac{1}{1+T}\right)}{\frac{1}{\beta}} = \frac{1}{1+T} \cong \frac{1}{T} \quad \text{for} \quad T >> 1$$

Gain Error Example: Noninverting Amplifier

- Problem: Find actual gain and gain error for an amplifier
- Given data: Ideal closed-loop gain of 200 (46 dB), open-loop gain of op amp is 10,000 (80 dB).
- Approach: Amplifier is designed to give ideal gain and deviations from ideal case are determined.
- *Note: R1 and R2* are not normally designed to compensate for finite open-loop gain of amplifier.
- Analysis:

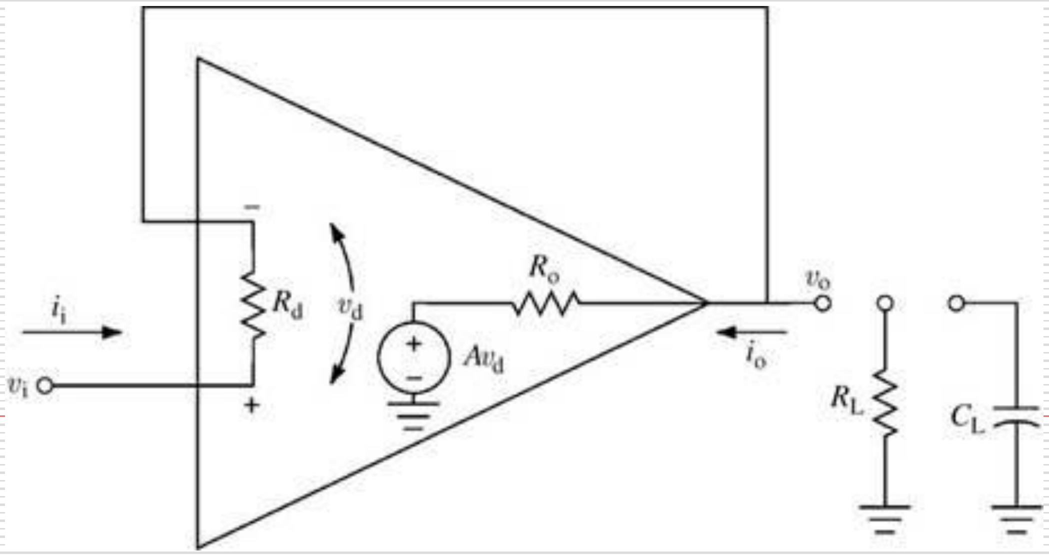
$$A_v = A_v^{Ideal}\left(\frac{T}{1+T}\right) \quad \text{with} \quad T = A\beta = 10^4\left(\frac{1}{200}\right) = 50$$

$$A_v^{Ideal} = \frac{1}{\beta} = 200 \qquad A_v = 200\left(\frac{50}{1+50}\right) = 196 \qquad FGE = \frac{200-196}{200} = 0.02$$

$$\text{Note:} \quad FGE \cong \frac{1}{T} = \frac{1}{50} = 0.02$$

Input Resistance

- Op-amp differential-input resistance R_d
  - For the FET-input 411, it is 1 TΩ
  - whereas for BJT-input op amps, it is about 2MΩ
  - Comparable to the value of some feedback resistors used



Voltage Follower Configuration

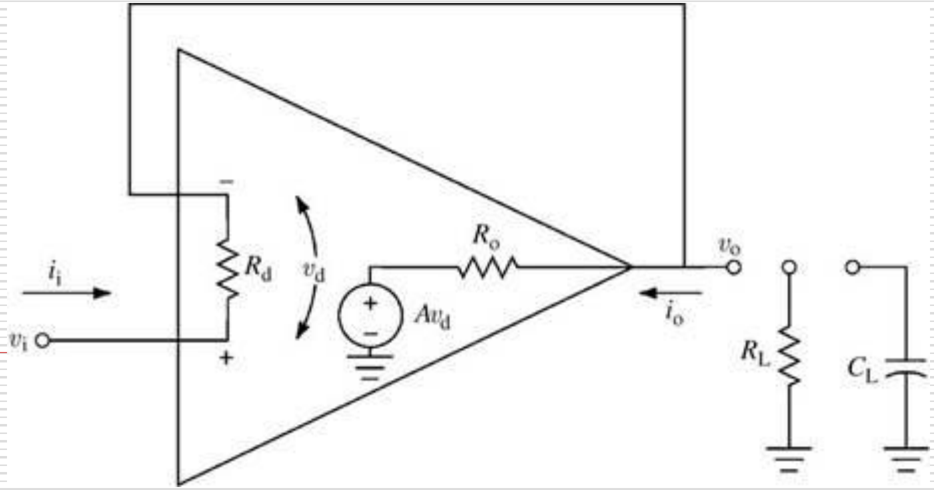
- In order to calculate the amplifier-circuit input resistance R_{ai}, assume a change in input voltage v_i.
- Amplifier-circuit input resistance R_{ai} is about $(10^5) \times (2\ M\Omega) = 200\ G\Omega$.
- The amplifier input impedance is much higher than the op-amp input impedance R_d.
- The amplifier output impedance is much smaller than the op-amp output impedance R_o.
- Noninverting amplifiers: R_{ai} is very high.
- Inverting amplifier: R_{ai} is usually small.

$$\Delta v_o = A\Delta v_d = A(\Delta v_i - \Delta v_o)$$

$$= \frac{A\Delta v_i}{A+1}$$

$$\Delta i_i = \frac{\Delta v_d}{R_d} = \frac{\Delta v_i - \Delta v_o}{R_d} = \frac{\Delta v_i}{(A+1)R_d}$$

$$R_{ai} = \frac{\Delta v_i}{\Delta i_i} = (A+1)R_d \cong AR_d$$



Input Resistance of Noninverting Amplifier

Test voltage source v_x is applied to input and current i_x is calculated:

$$i_x = \frac{v_x - v_1}{R_{id}} \quad \Big| \quad v_1 = i_1 R_1 \cong i_2 R_1$$

Assuming $i_- << i_2$ gives $i_1 \cong i_2$

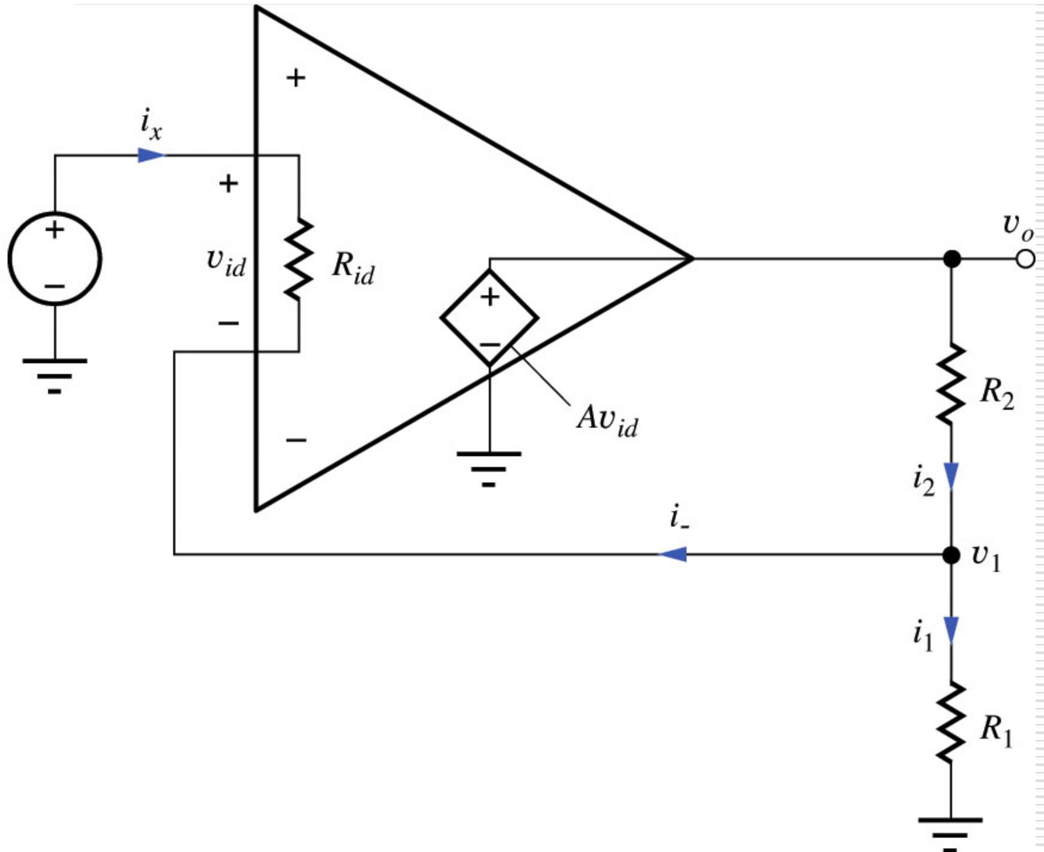
$$v_1 = \frac{R_1}{R_1 + R_2} v_o = \beta v_o$$

$$v_1 = \beta\left(A v_{id}\right) = A\beta\left(v_x - v_1\right)$$

$$v_1 = \frac{A\beta}{1 + A\beta} v_x = \frac{T}{1+T} v_x$$

$$\therefore \quad i_x = \frac{v_x - v_1}{R_{id}} = \frac{v_x - \frac{T}{1+T} v_x}{R_{id}} = \frac{v_x}{R_{id}\left(1+T\right)}$$

$$R_{in} = R_{id}\left(1+T\right)$$



OUTPUT RESISTANCE

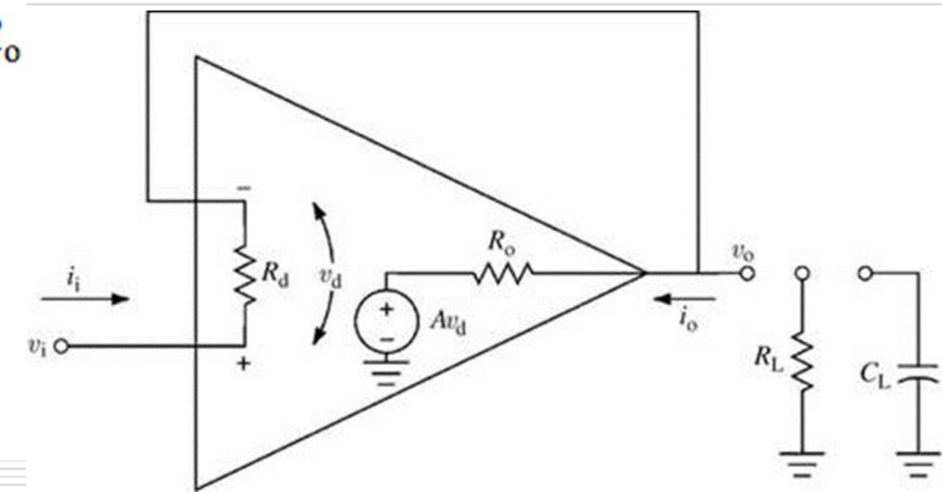
- The op-amp output resistance is about 40 Ω for the typical op amp, seems large for some applications.
- However, its value is usually not important because of the benefits of feedback.
- To calculate the amplifier-circuit output resistance Rao, assume that load resistor RL is attached to the output, causing a change in output current Δi_o.
- Because i_o flows through R_o, there is an additional voltage drop $\Delta i_o R_o$.

$$-\Delta v_d = \Delta v_o = A\Delta v_d + \Delta i_o R_o = -A\Delta v_o + \Delta i_o R_o$$

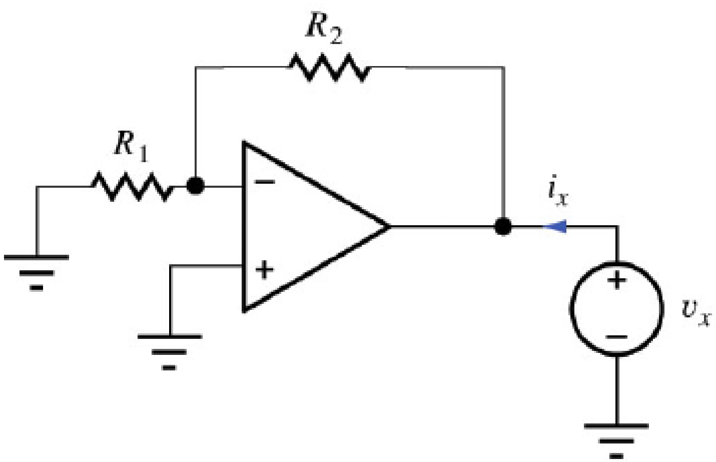
$$(A+1)\Delta v_o = \Delta i_o R_o$$

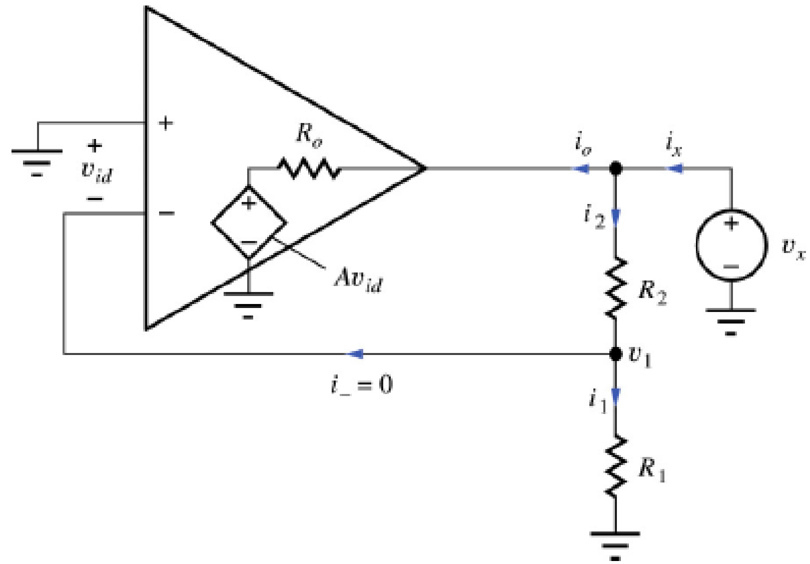
$$R_{ao} = \frac{\Delta v_o}{\Delta i_o} = \frac{R_o}{A+1} \cong R_o / A$$

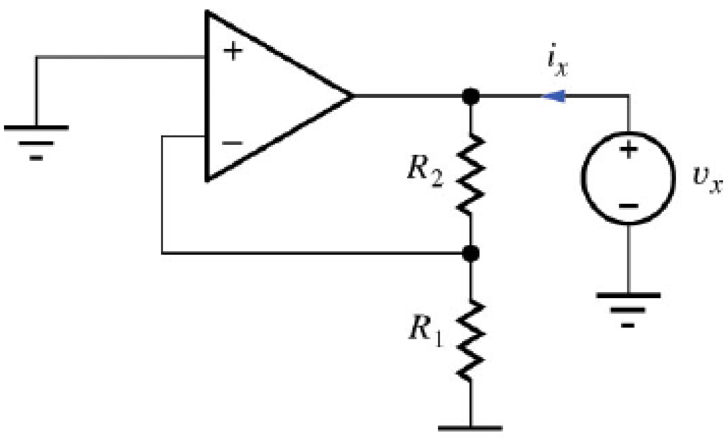
Thus the amplifier-circuit output resistance Rao is about 40/(105) = 0.0004 Ω, a value negligible in most circuits.



Output Resistance (Both Noninverting and Inverting Amplifiers)







The output terminal is driven by test source v_x and current i_x is calculated to determine output resistance (all independent sources are turned off). Note that the equivalent circuit is same for both inverting and non-inverting amplifiers.

$$R_{out} = \frac{v_x}{i_x}$$