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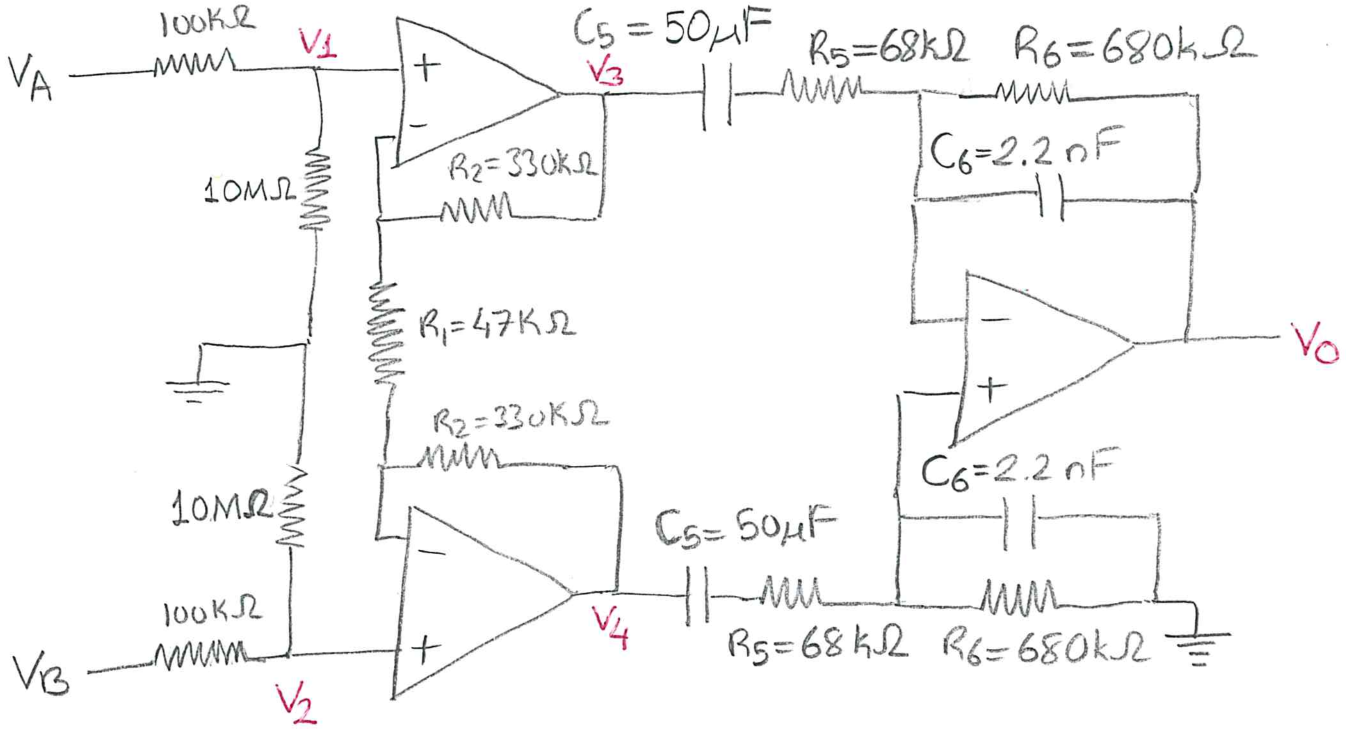
Q1)

Figure shows the circuit of an instrumentation amplifier used for recording electrocardiogram. The right arm of the patient is connected to point A, the left arm to point B and the right leg to the ground G. The common mode voltage arising out of the capacitative coupling of the patient to the ground, $V_{cm} = 10\,mV_{p\text{-}p}$, and a common mode rejection ratio of 78 dB is measured at 50Hz.

a) Sketch the frequency response of the circuit and calculate its differential gain at 50Hz.

b) It is specified that any unbalance in electrode contact impedances should not lead to more than 0.5 mV p-p error at output V_o. What is the maximum unbalance that the circuit can tolerate? By how much would the CMRR have to be increased for the specification to be met with a 3kΩ electrode unbalance?



ANSWER

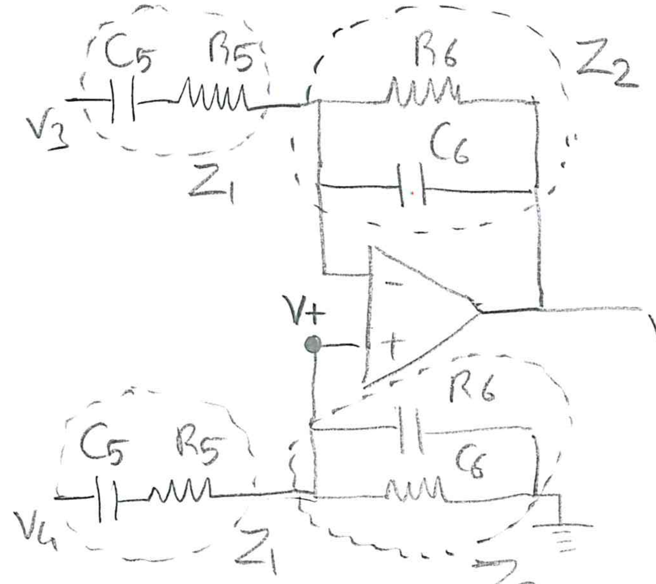
a) $V_1 = V_A \cdot \dfrac{10^7}{10^5+10^7} = 0.9901\,V_A \qquad V_2 = V_B \cdot \dfrac{10^7}{10^5+10^7} = 0.9901\,V_B$

$$\frac{V_3 - V_1}{R_2} = \frac{V_1 - V_2}{R_1}, \qquad \frac{V_2 - V_4}{R_2} = \frac{V_1 - V_2}{R_1}$$

$$\frac{V_3 - V_1}{R_2} + \frac{V_2 - V_4}{R_2} = \frac{2(V_1 - V_2)}{R_1}$$

$$\frac{V_3 - V_4}{R_2} = \frac{V_1 - V_2}{R_2} + 2\frac{(V_1 - V_2)}{R_1}$$

$$\frac{V_3 - V_4}{V_1 - V_2} = \left(\frac{1}{R_2} + \frac{2}{R_1}\right) R_2 = \frac{R_1 + 2R_2}{R_1} = 15.04 \; //$$



$$V_+ = \frac{V_4 \cdot Z_2}{Z_1 + Z_2}$$

$$\frac{V_3 - V_+}{Z_1} = \frac{V_+ - V_f}{Z_2}$$

$$-\left(\frac{V_3}{Z_1} - V_+\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)\right) Z_2 = V_0$$

$$-Z_2\left(\frac{V_3}{Z_1} - V^+\left(\frac{Z_2 + Z_1}{Z_1 Z_2}\right)\right) = V_0$$

substitute $V_+ = \frac{V_4 \cdot Z_2}{Z_1 + Z_2}$

$$-Z_2\left(\frac{V_3}{Z_1} - \frac{V_4 Z_2}{(Z_1 + Z_2)} \frac{(Z_1 + Z_2)}{Z_1 Z_2}\right) = V_0$$

$$Z_1 = R_5 + \frac{1}{jwC_5} = \frac{1 + jwR_5C_5}{jwC_5}$$

$$-Z_2\left(\frac{V_3}{Z_1} - \frac{V_4}{Z_1}\right) = V_0$$

$$Z_2 = \left(\frac{1}{R_6} + \frac{1}{1/jwC_6}\right)^{-1} = \frac{R_6}{1 + jwC_6R_6}$$

$$\frac{V_0}{V_4 - V_3} = \frac{Z_2}{Z_1} = \frac{jwC_5R_6}{(1 + jwR_5C_5)(1 + jwC_6R_6)}$$

This filter can be a passband or bandreject filter since it has two capacitors. In order to find the type of the filter check filter output at $w = 0$ and $w = \infty$

at $w = 0$ $\quad \frac{V_0}{V_4 - V_3} = \frac{0}{(1+0)(1+0)} = 0$

at $w = \infty$ $\quad \frac{V_0}{V_4 - V_3} = \frac{\infty}{\infty}$ then according to L'Hopital rule

take derivative of numerator and denominator

$$\frac{V_0}{V_4 - V_3} = \frac{jC_5R_6}{jR_5C_5 + jR_6C_6 - 2R_5C_5R_6C_6 w} = 0 \text{ as } w \to \infty$$

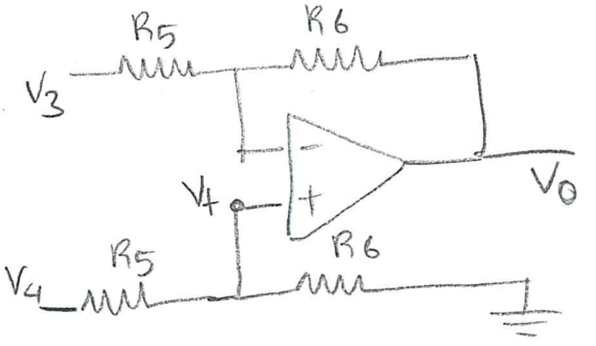
Then this filter must be a band pass filter

Filter cut-off frequencies are

$$f_{c_1} = \frac{1}{2\pi C_5 R_5} = \frac{1}{2\pi\, 50.10^{-6} .\, 68.10^{3}} = 0.047\,Hz$$

$$f_{c_2} = \frac{1}{2\pi C_6 R_6} = \frac{1}{2\pi\, 680\, 10^{3} .\, 2{,}2\, 10^{-9}} = 106.4\,Hz$$

Passband gain of this filter can be determined by removing capacitors and deriving the gain of remaining circuit as shown below



$$V_+ = \frac{V_4 R_6}{R_5 + R_6} \; , \quad \frac{V_3 - V_+}{R_5} = \frac{V_+ - V_o}{R_6}$$

$$-R_6\left(\frac{V_3}{R_5} - V_+\left(\frac{1}{R_5} + \frac{1}{R_6}\right)\right) = V_o$$

$$-R_6\left(\frac{V_3}{R_5} - V^+\left(\frac{R_6 + R_5}{R_5 R_6}\right)\right) = V_o$$

put $V_+ = \dfrac{V_4 R_6}{R_5 + R_6}$ in this equation

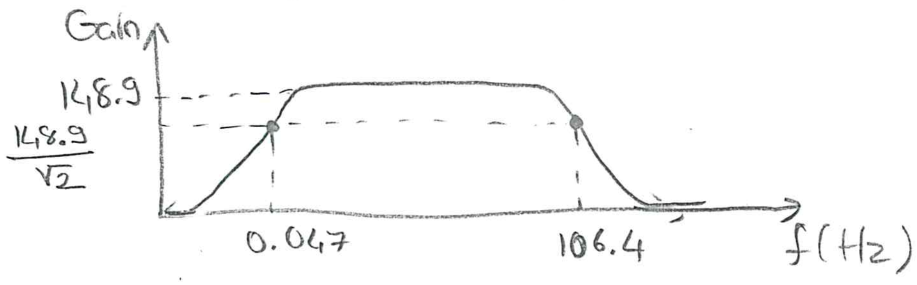
$$-R_6\left(\frac{V_3}{R_5} - \frac{V_4 R_6}{(R_5+R_6)}\frac{(R_6+R_5)}{R_6 R_5}\right) = V_o \; , \quad R_6\left(\frac{V_4 - V_3}{R_5}\right) = V_o$$

$$\frac{V_o}{V_4 - V_3} = \frac{R_6}{R_5} = \frac{680.10^{3}}{68.10^{3}} = 10$$

Total gain of instrumentation amplifier is

$$\frac{V_2 - V_1}{V_A - V_B} \cdot \frac{V_4 - V_3}{V_2 - V_1} \cdot \frac{V_o}{V_4 - V_3} = 0{,}9901 \cdot 15{,}04 \cdot 10 = 148.9 = \frac{V_o}{V_A - V_B}$$

The frequency response of the instrumentation amplifier is



b) The differential gain of the the circuit at 50Hz is 148.9 since this frequency is at the passband region. CMRR of this circuit at 50 Hz is given as

$$CMRR = 78dB = 20 \log_{10}\left(\frac{Ad}{Ac}\right) \begin{array}{l} \to \text{differential gain} \\ \to \text{common mode gain} \end{array}$$

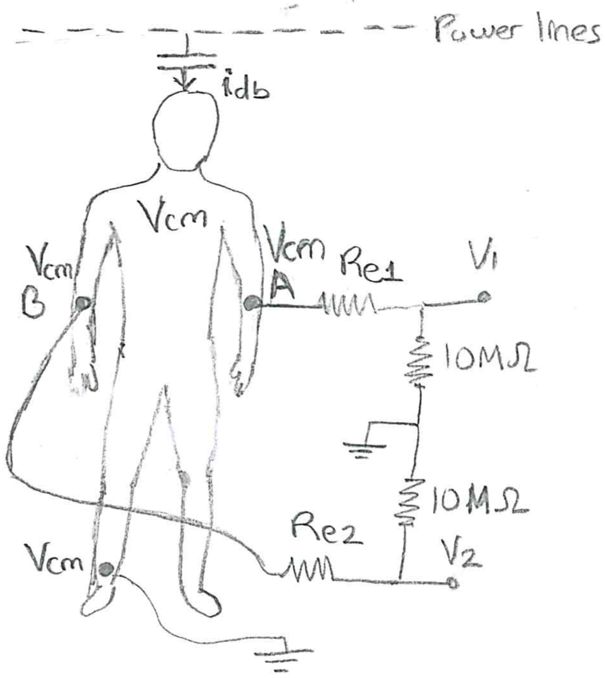
$$= 20 \log_{10} Ad - 20 \log_{10} Ac$$

$$78 = 20 \log_{10} 148.9 - 20 \log_{10} Ac$$

$$20 \log_{10} Ac = 43.5 - 78 = -34.5$$

$$Ac = 188.10^{-4} \quad \text{at } 50Hz$$

Noise at the output of instrumentation amplifier is an effect of common mode voltage Vcm which is caused by power lines. Vcm generates noise in two ways. in common mode and in differentral mode. As Vcm is the same at inputs A and B, instrumentation amplifier operates in common mode and Vcm is multiplied with common mode gain Ac and appears at ouput



$$n_{cm} = Ac\, Vcm = 188.10^{-4}.10.10^{-3} = 188mV$$

In the question electrode resistances are represented with $Re_1 = Re_2 = 100k\Omega$. If resistances of these electrodes are different from each other, the instrumentation amplifier operates at differential mode since the voltage at V_1 and V_2 is different. Let's assume the difference between resistances of electrodes is $Re_1 - Re_2 = d$

$$V_1 = Vcm \frac{10M\Omega}{Re_2 + d + 10M\Omega}$$

$$V_2 = Vcm \frac{10M\Omega}{Re_2 + 10M\Omega}$$

As $Re_2 = 100k\Omega$

$$V_2 - V_1 = Vcm \cdot \frac{10M\Omega}{100k\Omega + 10M\Omega} - \frac{10M\Omega}{100k\Omega + d + 10M\Omega} = \frac{d.10.10^6}{(10,1.10^6)(10.1.10^6 + d)} Vc$$

In addition to common mode error, n_{cm}, there will be differential mode error, n_{dm} which is equal to the multiplication of $V_2 - V_1$ with differential gain Ad.

$$n_{dm} = (V_2 - V_1) Ad = \frac{d.10.10^6}{(10,1.10^6)(10,1.10^6 + d)} 10.10^{-3}.148,9$$

The total noise, n is

$$n = n_{cm} + n_{dm}$$

The maximum acceptable error voltage is 0.5mV then

$$n_{cm} + n_{dm} = 0.5mV$$

$$n_{dm} = 0.5mV - 0.188mV = 0.312mV = \frac{d.10.10^6}{(10.1.10^6).(10{,}1.10^6+d)} 10.10^{-3}.148,$$

$$d = 2k\Omega$$

is the maximum allowable electrode impedance mismatch amount.

if there is $3k\Omega$ resistance unbalance then $d = 3k\Omega$

$$n_{dm} = \frac{3.10^3.\ 10.10^6}{(10{,}1.10^6).(10{,}1.10^6+3.10^3)} 148.9.10mV = 0.437mV$$

Not to exceed maximum acceptable error 0.5mV then n_{cm} should be

$$n_{cm} = 0.5mV - n_{dm} = 0.5 - 0.437 = 0.063mV$$

$n_{cm} = A_c V_{cm}$, then $A_c = \frac{n_{cm}}{V_{cm}} = \frac{0.063mV}{10mV} = 0.0063$

$$CMRR = 20\log_{10}\left(\frac{A_d}{A_c}\right) = 20\log_{10} A_d - 20\log_{10} A_c =$$

$$= 20\log_{10}(148.9) - 20\log_{10}(6.3\,10^{-3}) = 43.5 - (-44) = 87.5dB$$

An instrumentation amplifier having CMRR=87.5dB is required to compensate $3k\Omega$ resistance unbalance between electrode resistan

(Q2)

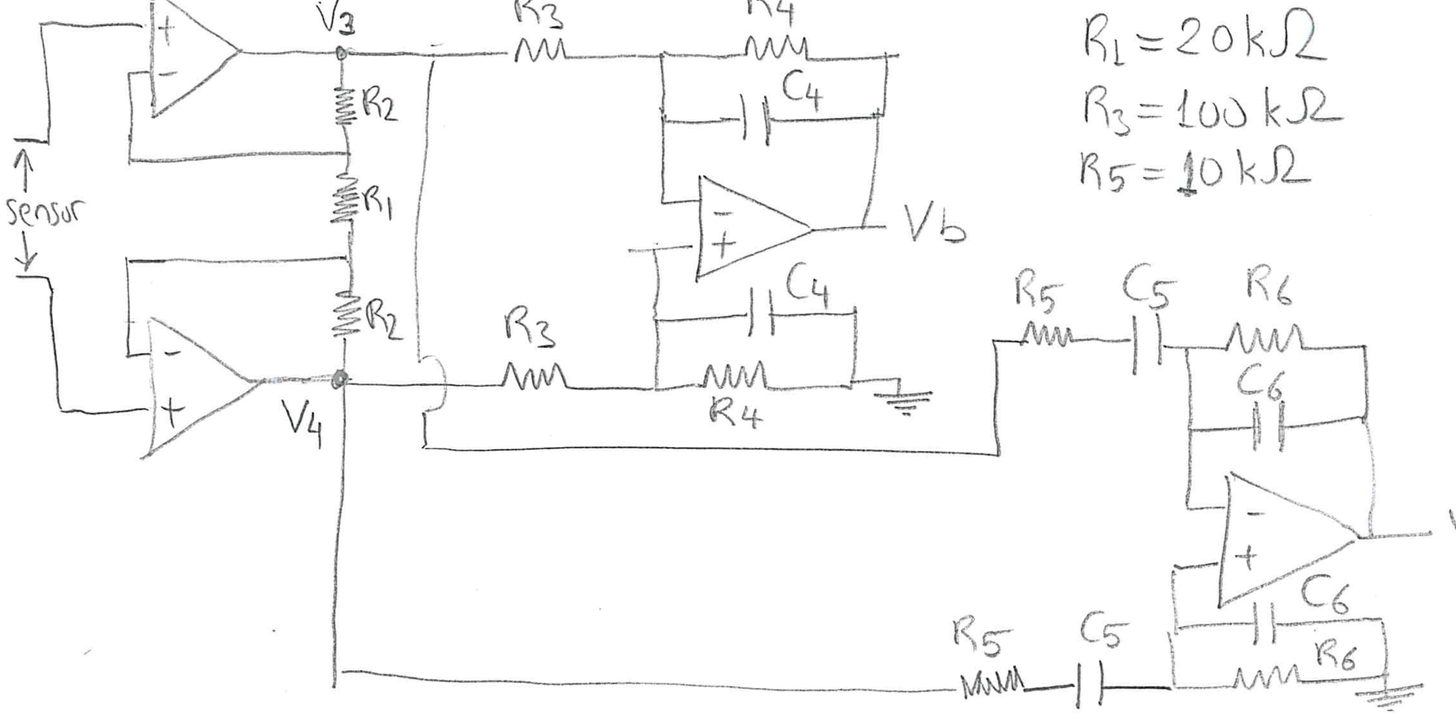
A patient is attached to an oscillometric blood pressure device. The circuit diagram for processing the output of the pressure sensor prior to digitisation is given below figure. The sensor which is attached to this circuit has a sensitivity of approximately 1mV per mmHg. The two outputs V_b and V_f are the baseline pressure signal and the fluctuation signal respectively.

a) Both output V_b and V_f are digitised by the same ADC which has 0-3V input range. Given that the observed fluctuations are up to 6 mmHg and cuff pressure is 300mmHg at maximum. What should be the gains for V_b and V_f respectively?

b) The op-amp circuits which are to be used have a maximum practicable gain of 25. What should be the gains for input op-amp circuit and for filters producing V_b and V_f respectively?

c) What must be the value of R_4? What is the type of filter generating V_b? If cut-off frequency of this filter is 0.5Hz, what is the value of C_4

d) The cut-off frequencies of the filter generating V_f is 0.5Hz and 30Hz. What is the type of this filter? What must be the values of R_6, C_5 and C_6? Assume that R_5 and C_5 determines low-cutoff frequency while R_6 and C_6 determines high cut-off frequency.



$R_1 = 20\,k\Omega$
$R_3 = 100\,k\Omega$
$R_5 = 10\,k\Omega$

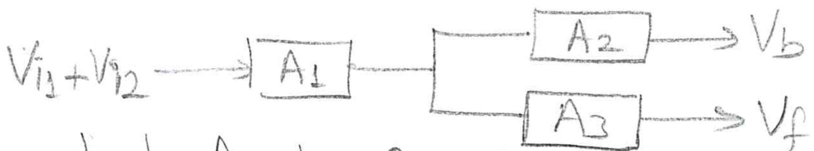
a) The input to the instrumentation amplifier is signal from pressure sensor which senses cuff pressure V_b and fluctuations V_f together. However they are separated at the output by filtering. First filter's output V_b is arised from V_{i1} resulting from cuff pressure, while second filter's output V_f arised from V_{i2} resulting from fluctuations. Both maximum amplitutes of V_b and V_f is 3V, then gains A_b and A_f of instrumentation amplifier for V_{i1} and V_{i2} are :

$$V_{i1\,max} = \text{Sensitivity} \cdot \text{Maximum Pressure} = 1\,\frac{mV}{mmHg} \cdot 300\,mmHg = 300mV$$

$$V_{i2\,max} = \text{Sensitivity} \cdot \text{Maximum Amplitude} = 1\,\frac{mV}{mmHg} \cdot 6mmHg = 6mV$$

$$A_b = \frac{V_{b\,max}}{V_{i1\,max}} = \frac{3V}{300\,mV} = 10 \,, \quad A_f = \frac{V_{f\,max}}{V_{i2\,max}} = \frac{3V}{6mV} = 500$$

b) if maximum gain of an op-amp circuit is 25, then gain of first op-amp circuit A_1 can be 20 or 25. The gain of filter for V_b is represented with A_2, the other filter's gain is represented with A_3.



Let A_1 be 20 then A_2 become more simplified.

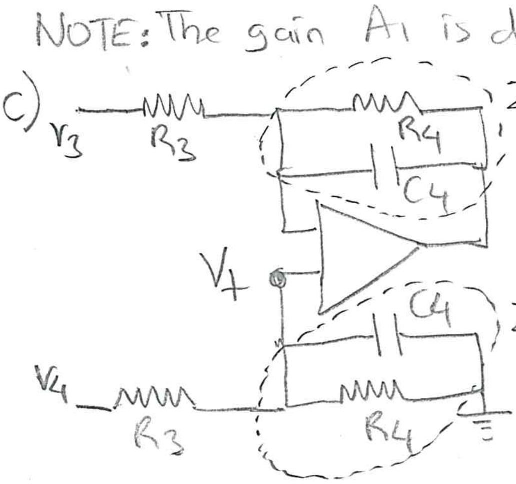
$A_1 = 20$ then $A_3 = \dfrac{500}{A_1} = 25$, $A_2 = \dfrac{10}{A_1} = 0.5$

If $A_1 = 20$, then the gain of the first circuit is

$$A_1 = \frac{R_1 + 2R_2}{R_1} \Rightarrow R_2 = \frac{A_1 R_1 - R_1}{2} = \frac{20.20 - 20}{2} = 190k\Omega$$

NOTE: The gain A_1 is derived in the previous question.

c)



$$\frac{V_4}{R_3 + Z_1} \cdot Z_1 = V^+ \qquad \frac{V_3 - V^+}{R_3} = \frac{V^+ - V_b}{Z_1}$$

$$-Z_1\left(\frac{V_3}{R_3} - \frac{V^+}{R_3} - \frac{V^+}{Z_1}\right) = V_b$$

$$-Z_1\left(\frac{V_3}{R_3} - V^+\left(\frac{Z_1 + R_3}{R_3 Z_1}\right)\right) = V_b$$

subsitute $\frac{V_4}{R_3+Z_1} \cdot Z_1 = V^+$, $-Z_1\left(\frac{V_3}{R_3} - \frac{V_4 Z_1}{R_3+Z_1} \cdot \frac{Z_1+R_3}{R_3 Z_1}\right) = Vb$

$$-Z_1\left(\frac{V_3 - V_4}{R_3}\right) = V_b \Rightarrow \frac{Vb}{V_4 - V_3} = \frac{Z_1}{R_3}$$

$$Z_1 = \left(\frac{1}{R_4} + \frac{1}{1/j\omega C_4}\right)^{-1} = \frac{R_4}{1+j\omega C_4 R_4} \Rightarrow \frac{Vb}{V_4 - V_3} = \frac{R_4}{R_3} \cdot \frac{1}{1+j\omega C_4 R_4}$$

when $w=0$, gain is $\frac{R_4}{R_3}$, when $w=\infty$ gain is 0 then this is a low pass filter.

$$A_2 = \frac{R_4}{R_3} = 0.5 \qquad R_3 = 100k\Omega \text{ then } 0.5 = \frac{R_4}{100k\Omega} \Rightarrow R_4 = 50k\Omega$$

$$f_c = 0.5Hz = \frac{1}{2\pi C_4 R_4} \text{ then } C_4 = \frac{1}{2\pi \cdot 0{,}5 \cdot 50 \cdot 10^3} = 6.36\, 10^{-6}F = 6.36\mu F$$

d) This filter is analyzed in the previous question

$$A_3 = \frac{V_f}{V_4 - V_3} = \frac{j\omega C_5 R_6}{(1+j\omega R_5 C_5)(1+j\omega C_6 R_6}$$

Passband gain is

$$A_3 = \frac{R_6}{R_5} = 25, \quad R_5 = 10k\Omega \text{ then } R_6 = A_3 R_5 = 25 \cdot 10 = 250k\Omega$$

$$f_{c_1} = \frac{1}{2\pi R_6 C_6} = \frac{1}{2\pi\, 250\, 10^3 C_6} = 30Hz \quad C_6 = \frac{1}{2\pi \cdot 250\, 10^3 \cdot 30} = 21nF$$

$$f_{c_2} = \frac{1}{2\pi R_5 C_5} = \frac{1}{2\pi \cdot 10 \cdot 10^3 C_5} = 0.5Hz \quad C_5 = \frac{1}{2\pi \cdot 10 \cdot 10^3 \cdot 0{,}5} = 31{,}8\mu F$$